

Study Guide

Polar and Rectangular Coordinates

Use the conversion formulas in the following examples to convert coordinates and equations from one coordinate system to the other.

Example 1 Find the rectangular coordinates of each point.

a. $P\left(3, \frac{3\pi}{4}\right)$

For $P\left(3, \frac{3\pi}{4}\right)$, $r = 3$ and $\theta = \frac{3\pi}{4}$.

Use the conversion formulas

$$x = r \cos \theta \text{ and } y = r \sin \theta.$$

$$\begin{aligned} x &= r \cos \theta & y &= r \sin \theta \\ &= 3 \cos \frac{3\pi}{4} & &= 3 \sin \frac{3\pi}{4} \\ &= 3\left(-\frac{\sqrt{2}}{2}\right) & &= 3\left(\frac{\sqrt{2}}{2}\right) \\ &\text{or } -\frac{3\sqrt{2}}{2} & &\text{or } \frac{3\sqrt{2}}{2} \end{aligned}$$

The rectangular coordinates of P are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, or $(-2.12, 2.12)$ to the nearest hundredth.

b. $Q(20, -60^\circ)$

For $Q(20, -60^\circ)$, $r = 20$ and $\theta = -60^\circ$.

$$\begin{aligned} x &= r \cos \theta & y &= r \sin \theta \\ &= 20 \cos (-60^\circ) & &= 20 \sin (-60^\circ) \\ &= 20(0.5) & &= 20\left(-\frac{\sqrt{3}}{2}\right) \\ &= 10 & &= -10\sqrt{3} \end{aligned}$$

The rectangular coordinates of Q are $(10, -10\sqrt{3})$, or approximately $(10, -17.32)$

Example 2 Find the polar coordinates of $R(5, -9)$.

For $R(5, -9)$, $x = 5$ and $y = -9$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & \theta &= \text{Arctan} \frac{y}{x} & x > 0 \\ &= \sqrt{5^2 + (-9)^2} & &= \text{Arctan} \frac{-9}{5} \\ &= \sqrt{106} \text{ or about } 10.30 & &\approx -1.06 \end{aligned}$$

To obtain an angle between 0 and 2π you can add 2π to the θ -value. This results in $\theta = 5.22$.

The polar coordinates of R are approximately $(10.30, 5.22)$.

Example 3 Write the polar equation $r = 5 \cos \theta$ in rectangular form.

$$\begin{aligned} r &= 5 \cos \theta \\ r^2 &= 5r \cos \theta & \text{Multiply each side by } r. \\ x^2 + y^2 &= 5x & r^2 = x^2 + y^2 \text{ and } r \cos \theta = x \end{aligned}$$

Practice

Polar and Rectangular Coordinates

Find the rectangular coordinates of each point with the given polar coordinates.

1. $(6, 120^\circ)$

2. $(-4, 45^\circ)$

3. $(4, \frac{\pi}{6})$

4. $(0, \frac{13\pi}{3})$

Find the polar coordinates of each point with the given rectangular coordinates. Use $0 \leq \theta < 2\pi$ and $r \geq 0$.

5. $(2, 2)$

6. $(2, -3)$

7. $(-3, \sqrt{3})$

8. $(-5, -8)$

Write each polar equation in rectangular form.

9. $r = 4$

10. $r \cos \theta = 5$

Write each rectangular equation in polar form.

11. $x^2 + y^2 = 9$

12. $y = 3$

13. Surveying A surveyor records the polar coordinates of the location of a landmark as $(40, 62^\circ)$. What are the rectangular coordinates?