2-7

Study Guide

Linear Programming

The following example outlines the procedure used to solve **linear programming** problems.

Example **The B & W Leather Company wants to add handmade belts and wallets to its product line. Each belt nets the company $18 in profit, and each wallet nets $12. Both belts and wallets require cutting and sewing. Belts require 2 hours of cutting time and 6 hours of sewing time. Wallets require 3 hours of cutting time and 3 hours of sewing time. If the cutting machine is available 12 hours a week and the sewing machine is available 18 hours per week, what mix of belts and wallets will produce the most profit within the constraints?**

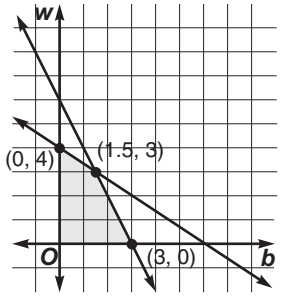
Define variables.

Let b = the number of belts.
Let w = the number of wallets.

Write inequalities.

$b \geq 0$
$w \geq 0$
$2b + 3w \leq 12$ cutting
$6b + 3w \leq 18$ sewing

Graph the system.



Write an equation.

Since the profit on belts is $18 and the profit on wallets is $12, the profit function is $B(b, w) = 18b + 12w$.

Substitute values.

$B(0, 0) = 18(0) + 12\ (0) = 0$
$B(0, 4) = 18(0) + 12(4) = 48$
$B(1.5, 3) = 18(1.5) + 12(3) = 63$
$B(3, 0) = 18(3) + 12(0) = 54$

Answer the problem.

The B & W Company will maximize profit if it makes and sells 1.5 belts for every 3 wallets.

When constraints of a linear programming problem cannot be satisfied simultaneously, then **infeasibility** is said to occur.

The solution of a linear programming problem is **unbounded** if the region defined by the constraints is infinitely large.

NAME _____ DATE _____ PERIOD _____

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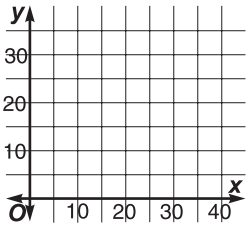
Practice

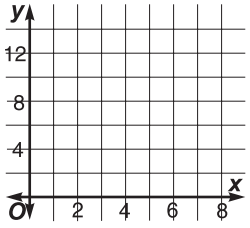
Linear Programming

Graph each system of inequalities. In a problem asking you to find the maximum value of f(x, y), state whether the situation is infeasible, has alternate optimal solutions, or is unbounded. In each system, assume that $x \geq 0$ and $y \geq 0$ unless stated otherwise.

1. $-2y \geq 2x - 36$
$x + y \geq 30$
$f(x, y) = 3x + 3y$

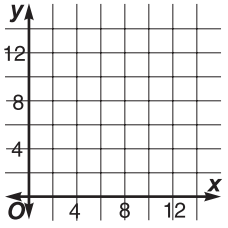


2. $2x + 2y \geq 10$
$2x + y \geq 8$
$f(x, y) = x + y$



Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

3. ***Nutrition*** A diet is to include at least 140 milligrams of Vitamin A and at least 145 milligrams of Vitamin B. These requirements can be obtained from two types of food. Type X contains 10 milligrams of Vitamin A and 20 milligrams of Vitamin B per pound. Type Y contains 30 milligrams of Vitamin A and 15 milligrams of Vitamin B per pound. If type X food costs \$12 per pound and type Y food costs \$8 per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?



4. ***Manufacturing*** The Cruiser Bicycle Company makes two styles of bicycles: the Traveler, which sells for \$200, and the Tourester, which sells for \$600. Each bicycle has the same frame and tires, but the assembly and painting time required for the Traveler is only 1 hour, while it is 3 hours for the Tourister. There are 300 frames and 360 hours of labor available for production. How many bicycles of each model should be produced to maximize revenue?