

ATHS FC – Math Department Al Ain Remedial Plan

Section		Date	
Name	Answers	Lesson	10.1, 10.2 and 10.3
ID		Marks	

Lesson 10.1 (Midpoint and Distance formula)

Question: 1

Find the midpoint of the line segment with endpoints at the given coordinates then find the distance between the points.

A (-2,-9), B (-6, 0)

$$\text{Midpt} = \left(\frac{-2-6}{2}, \frac{-9+0}{2} \right) = (-4, -4.5)$$

$$AB = \sqrt{(-6+2)^2 + (0+9)^2} = \sqrt{97} \text{ units}$$

Question: 2

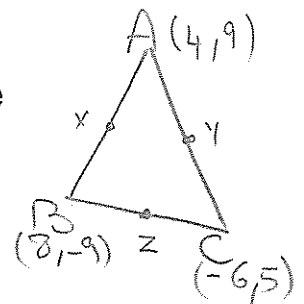
Triangle ABC has vertices A (4, 9), B (8, -9) and C (-6, 5)

- (a) Find the coordinates of the midpoint of each side of the triangle

$$X = \left(\frac{4+8}{2}, \frac{9-9}{2} \right) = (6, 0)$$

$$Y = \left(\frac{4-6}{2}, \frac{9+5}{2} \right) = (-1, 7)$$

$$Z = \left(\frac{8-6}{2}, \frac{-9+5}{2} \right) = (1, -2)$$



- (b) Find the perimeter of ABC and the perimeter of the triangle with vertices at the points found in part (a)

$$AB = \sqrt{(8-4)^2 + (-9-9)^2} = 18.4 \text{ units}$$

$$BC = \sqrt{(-6-8)^2 + (5+9)^2} \approx 19.8 \text{ units}$$

$$AC = \sqrt{(-6-4)^2 + (5-9)^2} \approx 10.8 \text{ units}$$

$$\text{Perimeter} = AB + BC + AC = 18.4 + 19.8 + 10.8 = 49 \text{ units}$$

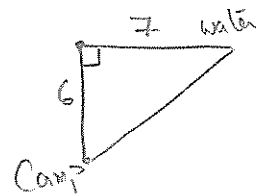
Question: 3

Carla and Sana left their campsite and hiked 6 miles directly north and then turned and hiked 7 miles east to view the waterfall.

- (a) How far is the waterfall from the campsite?

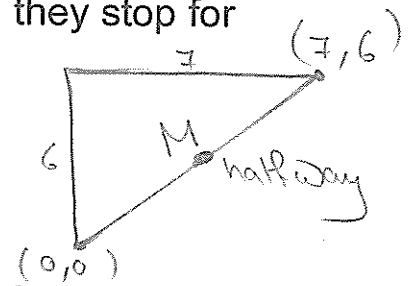
Using Pythagorean Theorem:

$$d = \sqrt{6^2 + 7^2} = \sqrt{85} = 9.2 \text{ miles}$$



- (b) If the campsite is located at the origin on a coordinate grid. At the waterfall they decide to head directly back to the campsite. If they stop halfway between the waterfall and the campsite for breakfast, at what coordinates will they stop for breakfast.

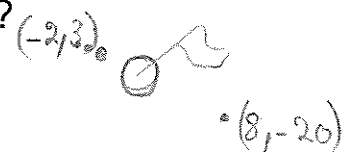
$$M = \left(\frac{7+0}{2}, \frac{6+0}{2} \right) = (3.5, 3)$$



Question: 4

Sara's disc is 20 feet ^{down} short and 8 feet ^{to the +ve} to the right of the basket. On his first putt, the disc lands 2 feet ^{to the -ve} to the left and 3 feet ^{to the -ve} beyond the basket. If the disc went in a straight line how far did it go?

$$d = \sqrt{(-2-8)^2 + (3+20)^2} = \sqrt{629} \approx 25 \text{ ft}$$



Lesson 10.2 (Parabola)

Try to remember the rules

Equations of a Parabola		
Form of Equations		
Direction of opening		
Vertex		
Axis of symmetry		
Focus		
Directrix		
Length of Latus Rectum		

$$\begin{aligned}
 y &= 2x^2 - 12x + 6 \\
 &= 2(x^2 - 6x) + 6 \\
 &= 2(x^2 - 6x + 3^2) + 6 - 18
 \end{aligned}$$

opposite sign

Question: 1

$$y = 2x^2 - 12x + 6$$

a) Identify direction of opening of the parabola.

upward

$$y = 2(x - 3)^2 - 12$$

b) Identify axis of symmetry.

$$x = h \Rightarrow x = 3$$

$$h = +3, k = -12$$

$$a = 2 > 0 \text{ upward}$$

c) Identify the vertex.

$$V(h, k) = V(3, -12)$$

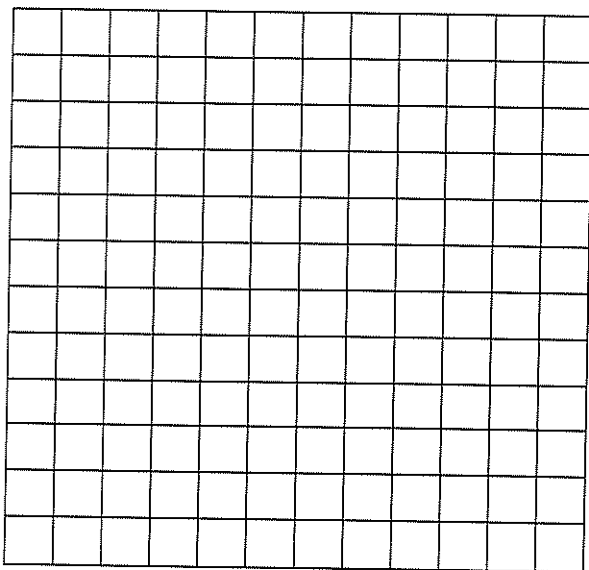
d) Identify the focus.

$$\text{Focus: } (h, k + \frac{1}{4a}) \quad \text{replace}$$

e) Identify the directrix.

$$y = k - \frac{1}{4a} \quad \text{replace}$$

f) Write the equation in standard form and Graph



Before graphing
you need to find
length of latus rectum
 \therefore length of latus rectum = $|\frac{1}{a}|$
 $= \frac{1}{2}$
then graph

$$x = 2y^2 + 4y + 6$$

← should be in order according to powers of y

Question: 2

$$x - 2y^2 = 4y + 6$$

a) Identify direction of opening of the parabola.

to the right

b) Identify axis of symmetry.

$$y = k \Rightarrow y = -1$$

c) Identify the vertex.

$$V(h, k) = V(4, -1)$$

d) Identify the focus.

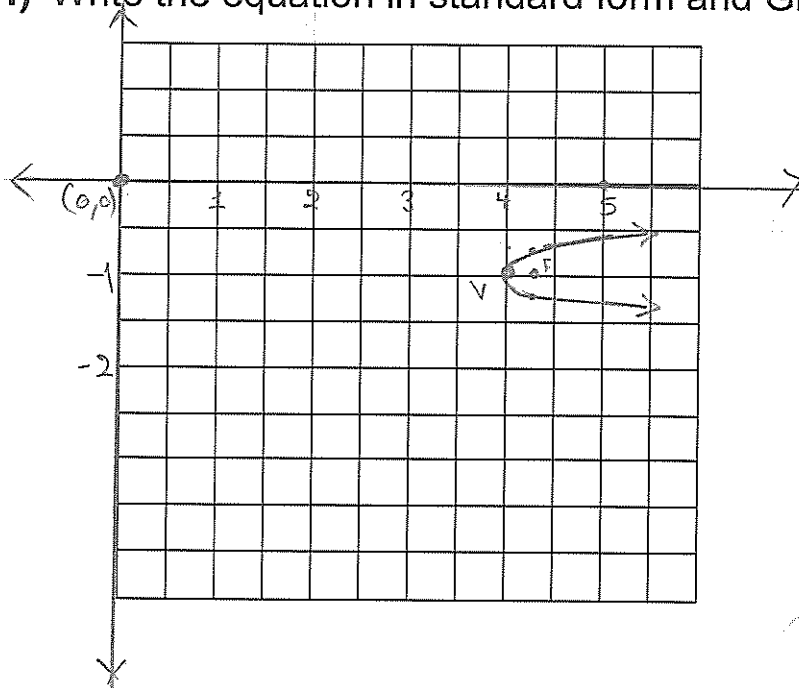
$$\text{Focus} = \left(h + \frac{1}{4a}, k \right)$$

$$\left(4 + \frac{1}{4(2)}, -1 \right) = (4.125, -1)$$

e) Identify the directrix.

$$x = h - \frac{1}{4a} = 3.875$$

f) Write the equation in standard form and Graph



length of latus rectum

$$= \left| \frac{1}{a} \right|$$

$$= \left| \frac{1}{2} \right| = \frac{1}{2}$$

Question: 3

$$y = -2(x + 4)^2 - 1$$

$$h = -4, k = -1$$
$$a = -2$$

a) Identify direction of opening of the parabola.

downwards

b) Identify axis of symmetry.

$$x = h \implies x = -4$$

c) Identify the vertex.

$$V(h, k) = (-4, -1)$$

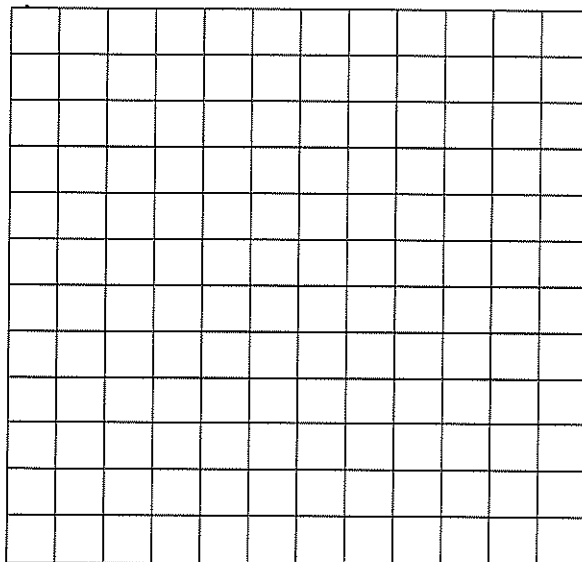
d) Identify the focus.

$$\text{Focus: } (h, k + \frac{1}{4a}) \quad (\text{replace})$$

e) Identify the directrix.

$$y = k - \frac{1}{4a} \quad (\text{replace})$$

g) Write the equation in standard form and Graph



$$\text{length of latus rectum} =$$
$$|\frac{1}{a}| = |-\frac{1}{2}|$$
$$= \frac{1}{2}$$

then graph

Question: 4

Write an equation for a parabola with vertex $(-2, -4)$ and directrix $y = 1$ then graph the equation

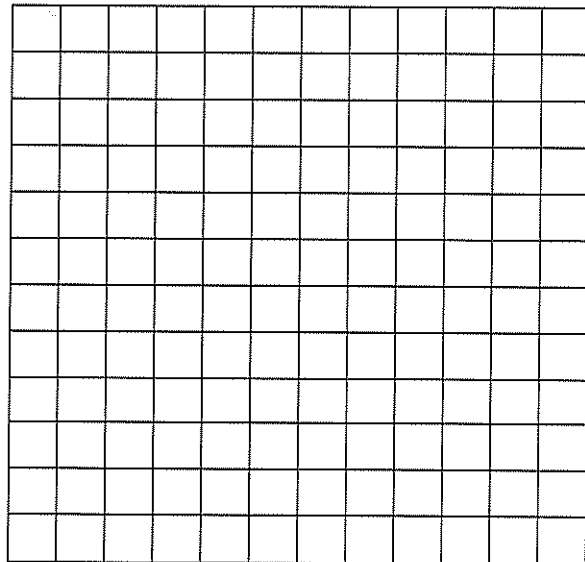
directrix: $y = 1 \Rightarrow$ it means it's a vertical parabola

$$\begin{aligned} \hookrightarrow \therefore y &= k - \frac{1}{4a} = 1 \\ -4 - \frac{1}{4a} &= 1 \\ -\frac{1}{4a} &= 5 \Rightarrow a = -\frac{1}{20} \end{aligned}$$

Equation of the Parabola

$$y = a(x-h)^2 + k$$

$$\begin{aligned} y &= -\frac{1}{20}(x-(-2))^2 + (-4) \\ &= -\frac{1}{20}(x+2)^2 - 4 \end{aligned}$$



Question: 5

then graph it

Write an equation for a parabola with vertex $(-2, 4)$ and directrix $x = -1$ then graph the equation

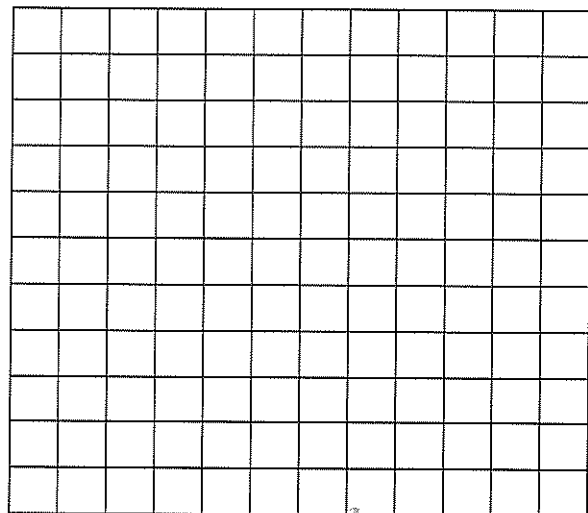
directrix: $x = -1 \Rightarrow$ It is a horizontal parabola

$$\begin{aligned} x &= h - \frac{1}{4a} = -1 \\ &= -2 - \frac{1}{4a} = -1 \\ -\frac{1}{4a} &= 1 \Rightarrow a = -\frac{1}{4} \end{aligned}$$

Equation of Parabola

$$x = a(y-k)^2 + h$$

$$x = -\frac{1}{4}(y-4)^2 - 2$$



Question: 6

Write an equation for a parabola with vertex $(1, 3)$, focus $(1, 5)$

Then graph the equation

(h, k) $(h, k + \frac{1}{4a})$
It is a vertical parabola

$$k + \frac{1}{4a} = 5 \Rightarrow 3 + \frac{1}{4a} = 5$$

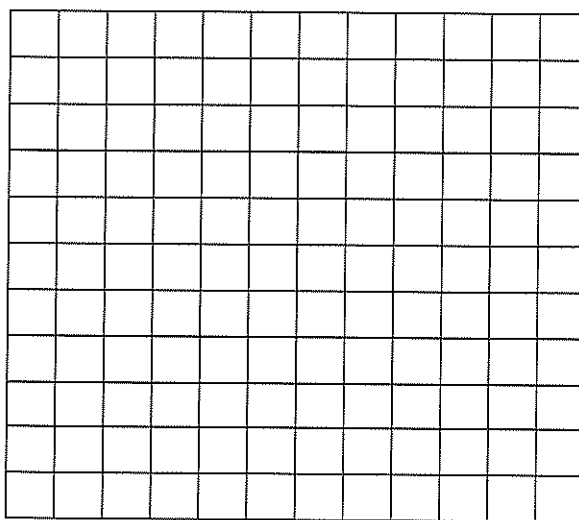
$$\frac{1}{4a} = 2 \Rightarrow a = \frac{1}{8}$$

Equation of the parabola

$$y = a(x-h)^2 + k$$

$$y = \frac{1}{8}(x-1)^2 + 3$$

then graph



Question: 7

Write an equation for a parabola with vertex $(-1, -5)$, focus $(-5, -5)$

Then graph the equation

(h, k) $(h + \frac{1}{4a}, k)$
It is a horizontal parabola

$$h + \frac{1}{4a} = -5$$

$$-1 + \frac{1}{4a} = -5$$

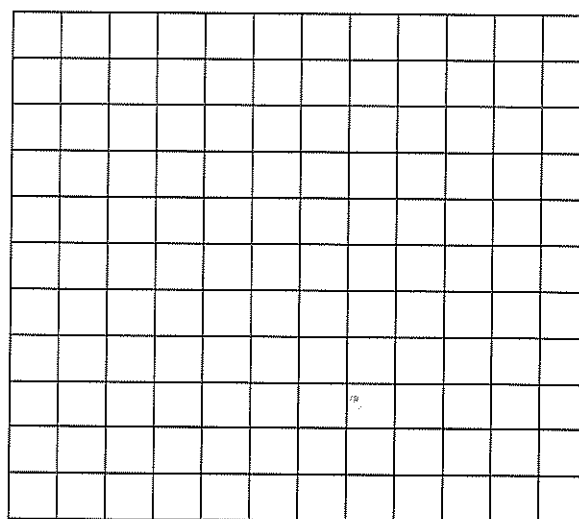
$$\frac{1}{4a} = -4 \Rightarrow a = -\frac{1}{16}$$

Equation of the parabola

$$x = a(y-k)^2 + h$$

$$x = -\frac{1}{16}(y+5)^2 - 1$$

then graph



Question: 8

When a ball is thrown, the path it travels is a parabola, suppose a baseball is thrown from ground level reaches a maximum height of 50 feet and hits the ground 200 feet from where it is thrown assuming this situation could be modeled on a coordinate plane with the focus of the parabola at the origin
Find the equation of the parabolic path of the ball

Find the answer in your notebooks
question 32 Pg 27

Question: 9

Solar energy may be harnessed by using parabolic mirrors. The mirrors reflect the rays from the sun to the focus of the parabola
The focus of each parabolic mirror at the facility described at the left is 6.25 feet above the vertex. The latus rectum is 25 feet long
Assume the focus is at the origin. Write the equation for the parabola formed by each mirror

The focus at the origin.
In order for the mirror to collect the Sun's energy, the parabola must open upward. So the vertex must be below the focus
Focus (0,0), vertex (0, -6.25).

latus rectum = $|\frac{1}{a}| = 25 \Rightarrow +\frac{1}{a} = 25 \Rightarrow a = \frac{1}{25}$

Question: 10

Write an equation of a parabola to model the shape of the suspension cable of the bridge shown. Assume that the origin is at the lowest point of the cables.

vertex (h, k)
(0, 0)

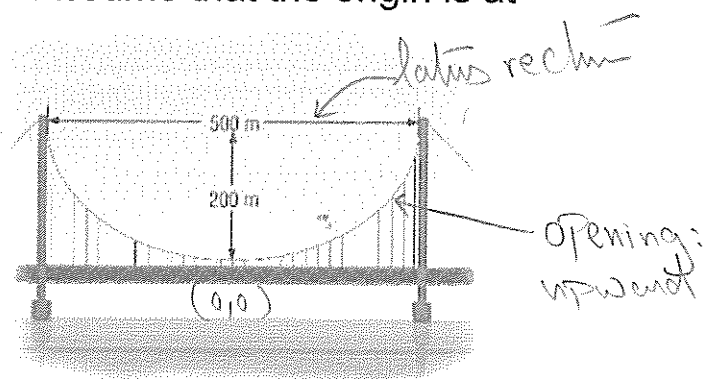
length of latus rectum = $|\frac{1}{a}|$

$500 = +\frac{1}{a}$

$\therefore a = \frac{1}{500}$

$y = a(x-h)^2 + k$

$= \frac{1}{500}(x-0)^2 + 0 \Rightarrow y = \frac{1}{500}x^2$



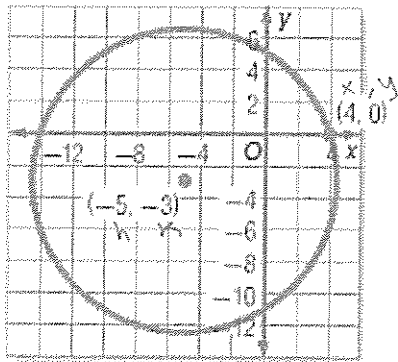
Lesson 10.3 (Circles)

Study the rules and complete the table.

Equations of a Circle		
Standard Form of Equation		
Center		
Radius		

Question 1: Write an equation for each Graph

(a)



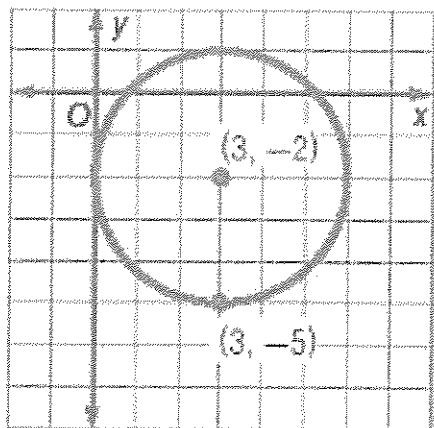
$$(x-h)^2 + (y-k)^2 = r^2$$

$$(4 - (-5))^2 + (0 - (-3))^2 = r^2$$

$$81 + 9 = r^2 \Rightarrow r^2 = 90$$

$$\boxed{(x+5)^2 + (y+3)^2 = 90}$$

(b)



Same way as the previous

Question 2: Write an equation for the circles that satisfies each set of conditions

- (a) Center $(-1, 6)$, radius 4 units

$$(x+1)^2 + (y-6)^2 = 4^2$$

$$(x+1)^2 + (y-6)^2 = 16$$

- (b) Endpoints of a diameter $(2, 5)$ and $(0, 0)$

length of diameter = $\sqrt{(2-0)^2 + (5-0)^2} = \sqrt{29} = 5.4$ units

radius = $\frac{d}{2} = \frac{5.4}{2} = 2.7$ units

Center = $(\frac{2+0}{2}, \frac{5+0}{2}) = (1, 2.5)$

$$(x-1)^2 + (y-2.5)^2 = (2.7)^2$$

- (c) Endpoints of a diameter $(4, -2)$ and $(-2, -6)$

Same as the Previous

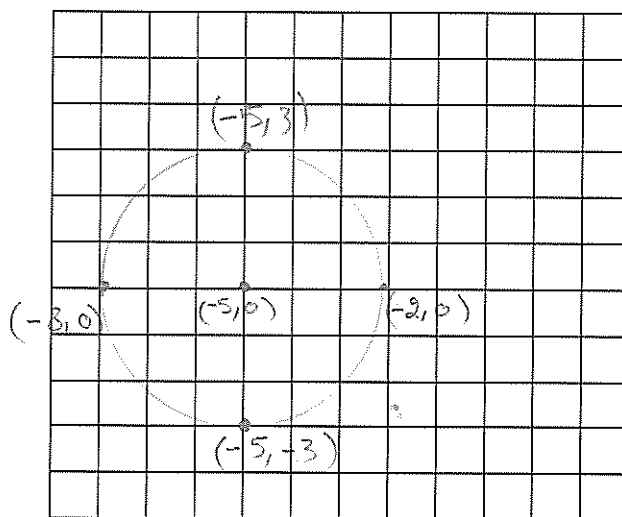
Question 3: Find the center and radius of each circle and then graph (table).

(a) $(x+5)^2 + y^2 = 9$

$$r^2 = 9 \Rightarrow r = 3$$

Center $(-5, 0)$

x	y
-2	0
-8	0
-5	3
-5	-3

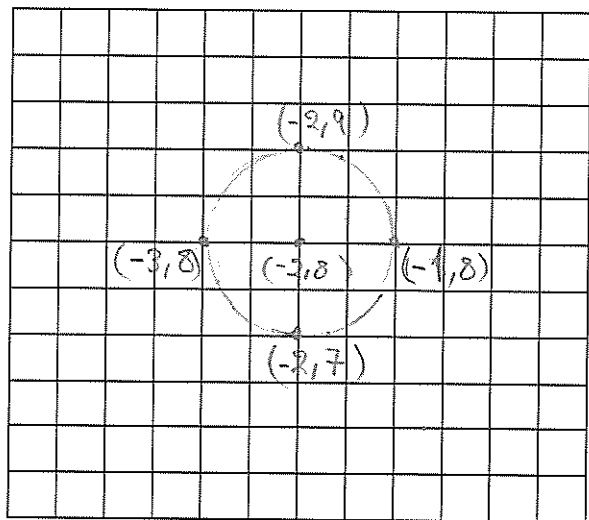


$$(b) (x+2)^2 + (y-8)^2 = 1$$

Center: $(-2, 8)$

radius = 1

x	y
-3	8
-1	8
-2	9
-2	7



$$(c) (x^2 + 4x) + (y^2 - 2y) - 11 = 0$$

$$(x^2 + 4x) + (y^2 - 2y) = 11$$

$$(x^2 + 4x + 2^2) + (y^2 - 2y + 1^2) - 4 - 1 = 11$$

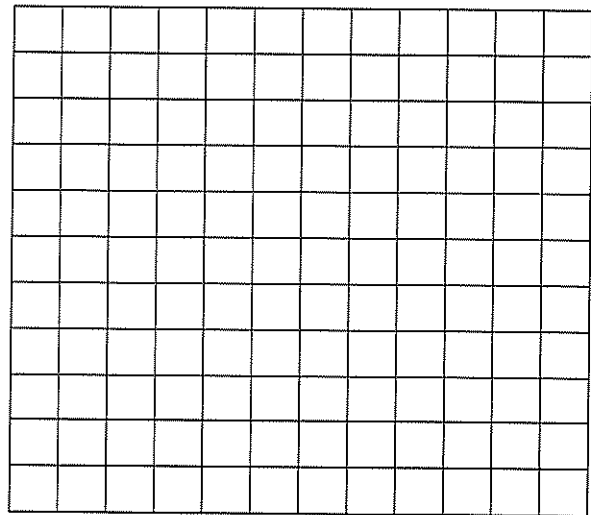
$$(x+2)^2 + (y+1)^2 = 11 + 5$$

$$(x+2)^2 + (y+1)^2 = 16$$

Center $(-2, -1)$

$$r^2 = 16 \rightarrow r = 4$$

then graph



Question 4: A sound loudspeaker in a school is located at point (65, 40). The speaker can be heard in a circle with radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

Center $(\overset{h}{65}, \overset{k}{40})$, radius = 100 ft

Equation 1-

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-65)^2 + (y-40)^2 = (100)^2$$

Question 5: Landscaping The plan for a park puts the center of a circular pond, of radius 0.6 miles, 2.5 miles east and 3.8 miles south of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin.

radius = 0.6 miles

the circular pond is centered at $(\overset{h}{2.5}, \overset{k}{-3.8})$

The equation is: $(x-h)^2 + (y-k)^2 = r^2$

$$(x-2.5)^2 + (y+3.8)^2 = (0.6)^2$$

$$(x-2.5)^2 + (y+3.8)^2 = 0.36$$