Applied Technology High School
Mathematics
Grade 12 Core
Worksheet: *calculus*

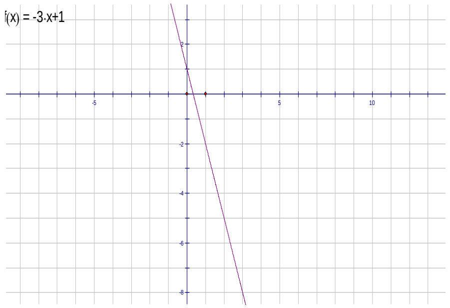
Lesson 12. 1 and 12.2

Question 1

Estimate $\lim_{x\to 2}(-3x+1)$ using a graph support your answer using a table .

Ans :

$\lim_{x\to 2}(-3x+1)$ = -5



By using the table :

x	1.9	1.99	1.999	2	2.001	2.01	2.1
F(x)	-4.7	-4.97	-4.997		-5.003	-5.03	-5.3

The table shows that when x get closer to 2 from left or from right , f(x) get closer to -5

Question 2

Evaluate each limit

1) $\lim\limits_{x\to4}\dfrac{x^2-9x+20}{x-4}=\lim\limits_{x\to4}\dfrac{(x-5)(x-4)}{x-4}=\lim\limits_{x\to4}(x-5)=4-5=-1$

2) $\lim\limits_{x\to3}\dfrac{3x^2-2x-21}{x-3}=\lim\limits_{x\to3}\dfrac{(3x+7)(x-3)}{x-3}=\lim\limits_{x\to3}(3x+7)=3(3)+7=16$

3) $\lim\limits_{x\to-3}\dfrac{x^2-9}{x^2+x-6}=\lim\limits_{x\to-3}\dfrac{(x-3)(x+3)}{(x-2)(x+3)}=\lim\limits_{x\to-3}\dfrac{(x-3)}{(x-2)}=\dfrac{-3-3}{-3-2}=\dfrac{6}{5}$

4) $\lim\limits_{x\to16}\dfrac{\sqrt{x}-4}{x-16}=\lim\limits_{x\to16}\dfrac{x-16}{(x-16)(\sqrt{x}+4)}=\lim\limits_{x\to16}\dfrac{1}{(\sqrt{x}+4)}=\dfrac{1}{\sqrt{16}+4}=\dfrac{1}{8}$

5) $\lim\limits_{x\to3}\dfrac{x^2}{2+\sqrt{x-3}}=\dfrac{3^2}{2+\sqrt{3-3}}=\dfrac{9}{2}$

6) $\lim\limits_{x\to-\infty}(x^5-6x+1)=\lim\limits_{x\to-\infty}x^5=(-\infty)^5=-\infty$

7) $\lim\limits_{x\to\infty}(2x^4+5x^2)=\lim\limits_{x\to\infty}2x^4=2(\infty)^4=\infty$

8) $\lim\limits_{x\to\infty}\dfrac{5x^4+2x^3-1}{2x^3+x^2-1}=\lim\limits_{x\to\infty}\dfrac{5x^4}{2x^3}=\infty$

$$9)\ \lim_{x\to\infty}\frac{2x^3+x^2-1}{5x^4+2x^3-1}=\lim_{x\to\infty}\frac{2x^3}{5x^4}=0$$

$$10)\ \lim_{x\to\infty}\frac{2x^4+x^2-1}{5x^4+2x^3-1}=\lim_{x\to\infty}\frac{2x^4}{5x^4}=\frac{2}{5}$$

$$13)\ \lim_{x\to 3^+}\frac{3-x}{|x-3|}=\lim_{x\to 3^+}\frac{3-x}{x-3}=-1$$

$$14)\ \lim_{x\to 3^-}\frac{3-x}{|x-3|}=\lim_{x\to 3^+}\frac{3-x}{3-x}=1$$

$$15)\ \lim_{n\to\infty}a_n=\frac{3n+1}{n+5}=\lim_{n\to\infty}\frac{3}{1}=3$$

Question 3

$$g(x)=\begin{cases}x+1 & if\ x<1\\ -x+5 & if\ x>1\\ 3 & if\ x=1\end{cases}$$

Estimate each one sided or two- sided limit, if it exists

$\lim_{x\to 1^-} g(x)$, $\lim_{x\to 1^+} g(x)$ and $\lim_{x\to 1} g(x)$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} x+1 = 1+1 = 2$

$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} -x+5 = -1+5 = 4$

$\lim_{x\to 1} g(x)$ *does not exist*

Question 4

Estimate each one sided or two- sided limit, if it exists

a) $\lim_{x\to 0^-} \frac{x}{|x|}$, $\lim_{x\to 0^+} \frac{x}{|x|}$ and $\lim_{x\to 0} \frac{x}{|x|}$

b) $\lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{-x} = -1$

c) $\lim_{x\to 0^+} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{x} = 1$

d) $\lim_{x\to 0} \frac{x}{|x|}$ *does not exist*

e) $\lim_{x\to 0} f(x)$ *where* $f(x) = \begin{cases} x-5 & if\ x<0 \\ x^2 & if\ x \geq 0 \end{cases}$

$\lim_{x\to 0^-} x-5 = -5$

$\lim_{x\to 0^+} x^2 = 0$

$\lim_{x\to 0^-} \neq \lim_{x\to 0^+}$

$\lim_{x\to 0} f(x) = DNE$

f)

$$\lim_{x \to \frac{\pi}{3}} \frac{\tan x}{x}$$

Ans

$$\lim_{x \to \frac{\pi}{3}} \frac{\tan x}{x} = \frac{\tan \frac{\pi}{3}}{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}}{\frac{\pi}{3}}$$

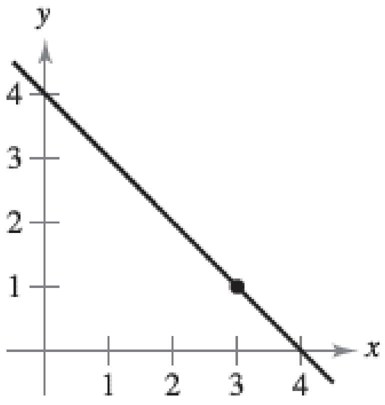
$$= \frac{3\sqrt{3}}{\pi}$$

The limit of $\frac{\tan x}{x}$ as x approaches $\frac{\pi}{3}$ is $\frac{3\sqrt{3}}{\pi}$.

Question 5

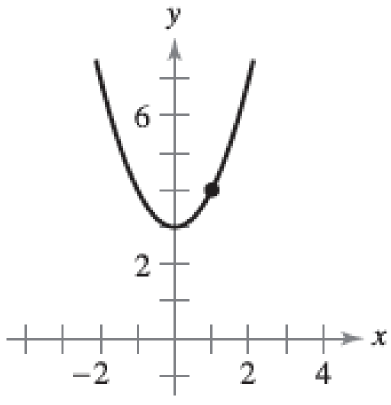
Use the graph to find the limits.

1. $\lim_{x \to 3} (4 - x)$



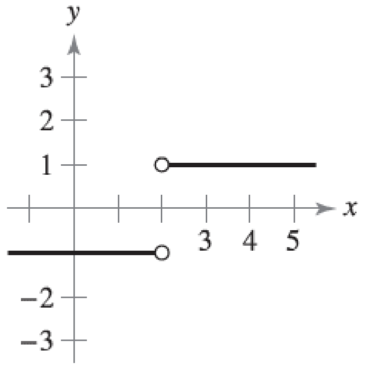
$\lim_{x\to 3}(4 - x) = 1$

2. $\lim_{x \to 1} (x^2 + 3)$



$lim_{\ x\to 1}(x^2 + 3) = 4$

3. $\lim_{x \to 2} \frac{|x - 2|}{x - 2}$



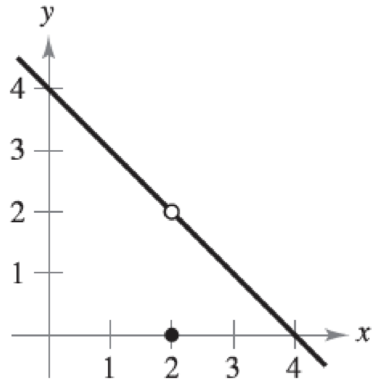
$\lim_{x\to 2^+}(\frac{|x-2|}{x-2}) = 1$

$\lim_{x\to 2^-}(\frac{|x-2|}{x-2}) = -1$

$\lim_{x\to 2}(\frac{|x-2|}{x-2}) = does\ not\ exist$

4. $\lim_{x\to 2} f(x)$

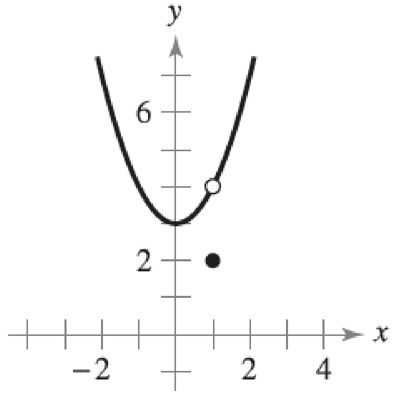
$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$



$\lim_{x\to 2} f(x) = 2$

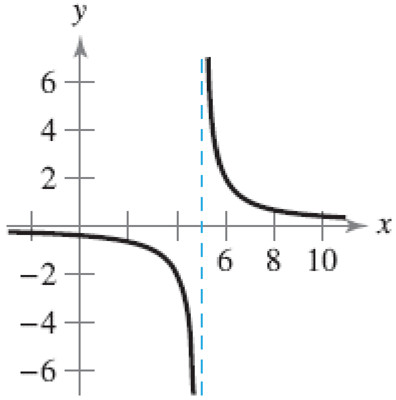
5. $\lim_{x\to 1} f(x)$

$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



$\lim_{x\to 1} f(x) = 4$

6. $\lim_{x\to 5} \frac{2}{x-5}$

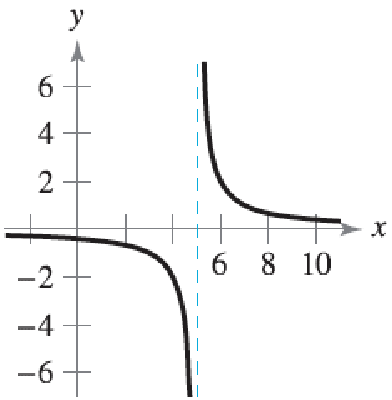


Does not Exist

$\lim_{x\to 5^-} \frac{2}{x-5} = -\infty$

$\lim_{x\to 5^+} \frac{2}{x-5} = \infty$

7. $\lim_{x \to 5} \frac{2}{x-5}$

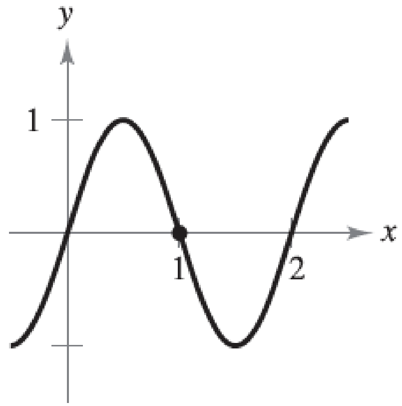


Does not Exist

$\lim_{x \to 5^-} \frac{2}{x-5} \;=\; -\infty$

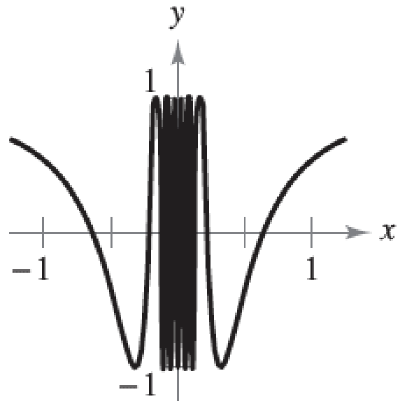
$\lim_{x \to 5^+} \frac{2}{x-5} \;=\; \infty$

8. $\lim_{x \to 1} \sin \pi x$



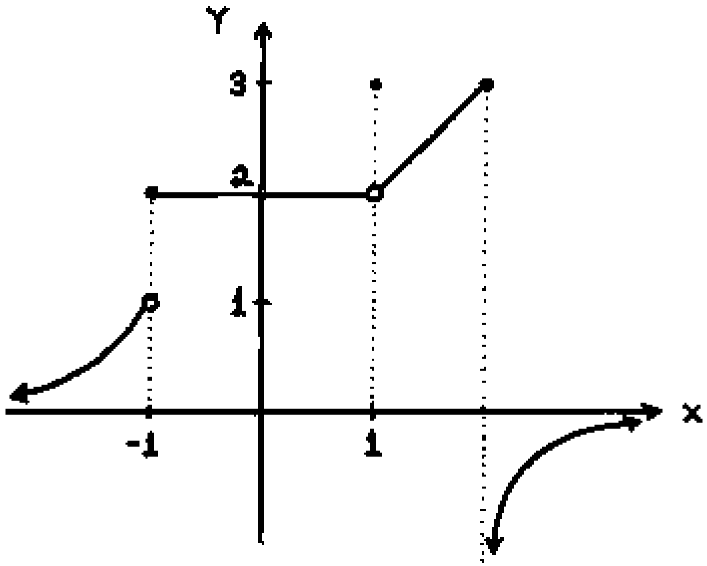
$\lim_{x \to 1} sin\pi x = 0$

9. $\lim_{x \to 0} \cos \frac{1}{x}$



$\lim_{x \to 0} cos\frac{1}{x} = Does\ not\ exist$

Question 6

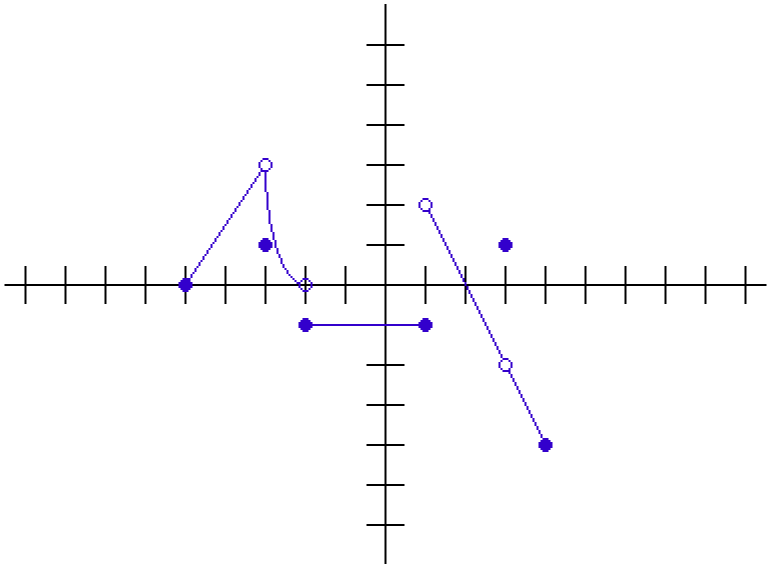


Determine the following limits

1. $\lim_{x\to -1^+} f(x) = 2$
2. $\lim_{x\to -1^-} f(x) = 1$
3. $\lim_{x\to -1} f(x) = does\ not\ exist$
4. $\lim_{x\to 1^+} f(x) = 2$
5. $\lim_{x\to 1^-} f(x) = 2$

6. $\lim_{x \to 1} f(x) = 2$
7. $\lim_{x \to 2^+} f(x) = -\infty$
8. $\lim_{x \to 2^-} f(x) = 3$
9. $\lim_{x \to 2} f(x) = does\ not\ exist$

Question 7 (a)



Determine the following limits

1. $\lim_{x \to 1^+} f(x) = 2$
2. $\lim_{x \to 1^-} f(x) = -1$
3. $\lim_{x \to 1} f(x) = does\ not\ exist$
4. $\lim_{x \to -2^+} f(x) = -1$
5. $\lim_{x \to -2^-} f(x) = 0$
6. $\lim_{x \to -2} f(x) = does\ not\ exist$
7. $\lim_{x \to 3} f(x) = -2$

8. $\lim_{x \to 1} f(x) = does\ not\ exist$

9. $\lim_{x \to 0} f(x) = -1$

10. $\lim_{x \to -3} f(x) = 3$

Question 7 (b)

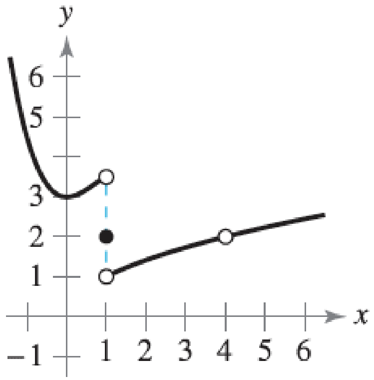
Use the graph of the function *f* to decide whether the value of the given quantity exists. If it does, find it. If not explain why.

(a) $f(1)$ **= 2**

(b) $\lim_{x \to 1} f(x)$ **= DNE**

(c) $f(4)$ **= Undefined**

(d) $\lim_{x \to 4} f(x)$ **= 2**

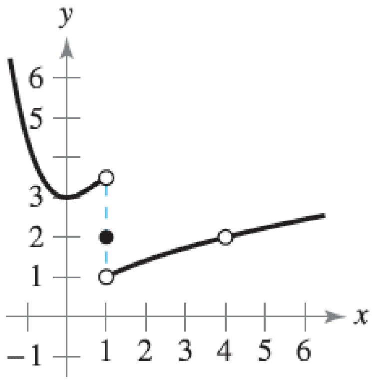


(a) $f(1)$

(b) $\lim_{x \to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x \to 4} f(x)$



11. (a) $f(-2)$ **= Undefined**

(b) $\lim_{x \to -2} f(x)$ **= DNE**

(c) $f(0)$ **= 4**

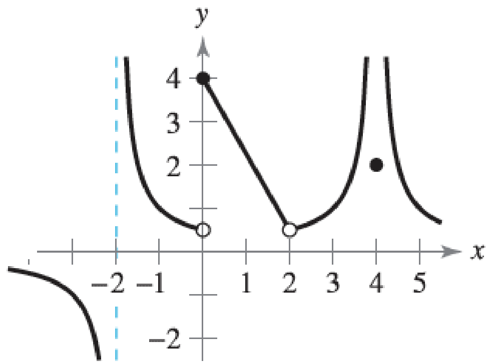
(d) $\lim_{x \to 0} f(x)$ **= DNE**

(e) $f(2)$ **= Undefined**

(f) $\lim_{x \to 2} f(x)$ **= 0.5**

(g) $f(4)$ **= 2**

(h) $\lim_{x \to 4} f(x)$ **= ∞**



ANS: a)2 , b) DNE , c) undefined , d) 2

Lesson 12. 4

Question 8

Find the derivative of each function

$a)\ f(x) = 4x^2 + 9x$

$\boldsymbol{f'(x) = -8x + 9}$

$b)\ f(x) = 4x^{\frac{3}{4}} - 8x^{\frac{1}{2}} + 5$

$\boldsymbol{f'(x) = 4\left(\frac{3}{4}\right)x^{\frac{-1}{4}} - 8\left(\frac{1}{2}\right)x^{\frac{-1}{2}} + 0}$

$\boldsymbol{f'(x) = 3x^{\frac{-1}{4}} - 4x^{\frac{-1}{2}} + 0}$

$c)\ f(x) = -3\sqrt[5]{x^6} = -3x^{\frac{6}{5}}$

$\boldsymbol{f'(x) = -3\left(\frac{6}{5}\right)x^{\frac{1}{5}} = \frac{-18}{5}x^{\frac{1}{5}}}$

$d)\ f(x) = \frac{1}{x^4} = x^{-4}$

$\boldsymbol{f'(x) = -4x^{-5}}$

$e)\ f(x) = \frac{x^2+8}{x^3-2}$

$\boldsymbol{f'(x) = \frac{(2x)(x^3-2)-(3x^2)(x^2+8)}{(x^3-2)^2}}$

$\boldsymbol{f'(x) = \frac{2x^4-4x-3x^4-24x^2}{(x^3-2)^2}}$

$\boldsymbol{f'(x) = \frac{-x^4-24x^2-4x}{(x^3-2)^2}}$

$f)\ f(x) = (x^2 - 4)(2x - 5)$

$\boldsymbol{f'(x) = 2x(2x - 5) + 2(x^2 - 4)}$

$\boldsymbol{f'(x) = 4x^2 - 10x + 2x^2 - 8 = 6x^2 - 10x - 8}$

Question 9

Find the equation of tangent at the given point

$f(x) = -3x^2 - 2x + 4 \quad ,(1,-1)$

$\boldsymbol{f'(x) = -6x - 2}$

$\boldsymbol{f'(1) = -6(1) - 2 = -8}$

$\boldsymbol{y = m(x - x_1) + y_1}$

$\boldsymbol{y = -8(x - 1) + (-1)}$

$\boldsymbol{y = -8x + 8 - 1}$

$\boldsymbol{y = -8x + 7}$

Question 10

The **velocity** in meters per second of a particle moving along a straight line is given by the function

$v(t) = 3t^2 - 6t + 5$

Where t is the time in seconds

Find the acceleration of the particle after 5 seconds

$\boldsymbol{a(t) = v'(t) = 6t - 6}$

$\boldsymbol{a(5) = v'(5) = 6(5) - 6 = 24m/s^2}$

Question 11

Evaluate limits to find the derivative of $f(x) = 4x^2 - 3$,then evaluate the derivative at x= 2 and -1

Definition of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - 3 - (4x^2 - 3)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{4(x^2 + \ xh + h^2) \quad - 3 - (4x^2 - 3)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{4x^2 + 4xh + 4h^2 - 3 - (4x^2 - 3)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\not{h}(4x + 4h)}{\not{h}}$$

$$f'(x) = \lim_{h \to 0} 4x + 4h$$

$= 4x + 4(0) = 4x$

$f'(2) = 4x = 4(2) = 8 \qquad and\ f'(-1) = 4x = 4(-1) = -4$

Lesson 12.6

Question 12

Find all antiderivatives for each function

a) $f(x) = 8x^3 + 5x^2 - 9x + 3$

$$F(x) = \frac{8x^4}{4} + \frac{5x^3}{3} - \frac{9x^2}{2} + 3x + c$$

$$F(x) = 2x^4 + \frac{5x^3}{3} - \frac{9x^2}{2} + 3x + c$$

b) $f(x) = \frac{3}{4}x^{\frac{2}{5}} + \frac{5}{8}x^{\frac{1}{3}} + x^{\frac{1}{2}}$

$$F(x) = \frac{3x^{\frac{7}{5}}}{4(\frac{7}{5})} + \frac{5x^{\frac{4}{3}}}{8(\frac{4}{3})} + \frac{x^{\frac{3}{2}}}{(\frac{3}{2})} + c$$

$$F(x) = \frac{3x^{\frac{7}{5}}}{(\frac{28}{5})} + \frac{5x^{\frac{4}{3}}}{(\frac{32}{3})} + \frac{x^{\frac{3}{2}}}{(\frac{3}{2})} + c$$

$$F(x) = \frac{15x^{\frac{7}{5}}}{28} + \frac{15x^{\frac{4}{3}}}{32} + \frac{2x^{\frac{3}{2}}}{3} + c$$

c) $f(x) = \frac{2}{x^4} = 2x^{-4}$

$\boldsymbol{F(x) = \frac{2x^{-3}}{-3} + c}$

d) $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} = \boldsymbol{F(x) = \frac{x^{\frac{4}{3}}}{(\frac{4}{3})} + c = \frac{3x^{\frac{4}{3}}}{4} + c}$

Lesson 12.6

<u>Question 13</u>

Evaluate each integral

a) $\int_{-3}^{-1}(x^3 + 8x^2 + 21x)dx$

$\boldsymbol{\frac{x^4}{4} + \frac{8x^3}{3} + \frac{21x^2}{2}}$

$\boldsymbol{\left[\frac{(-1)^4}{4} + \frac{8(-1)^3}{3} + \frac{21(-1)^2}{2}\right] - \left[\frac{(-3)^4}{4} + \frac{8(-3)^3}{3} + \frac{21(-3)^2}{2}\right] =}$

b) $\int_{-2}^{-1}(\frac{x^5}{2} + \frac{5x^4}{4})\, dx$

$\boldsymbol{\frac{x^6}{2(6)} + \frac{5x^5}{4(5)}}$

$$\frac{x^6}{12}+\frac{x^5}{4}$$

$$\left[\frac{(-1)^6}{12}+\frac{(-1)^5}{4}\right]-\left[\frac{(-2)^6}{12}+\frac{(-2)^5}{4}\right]$$

Question 14

Students in a technology class are participating in an egg drop competition
Each team of students must build a protective device that will keep an egg from
cracking after a 30 foot drop , the instanteneousd velocity is given by
$v(t) = -32t$, *t is the time in seconds*

a)*Find the position function* $s(t)$*of a dropped egg*

$$s(t) = \int -32t\, dt$$
$$s(t) = \frac{-32t^2}{2} + c$$
$$s(t) = -16t^2 + c$$
$$30 = -16(0)^2 + c$$
$$c = 30$$
$$s(t) = -16t^2 + c$$
$$s(t) = -16t^2 + 30$$

b)*for how long it will take for the egg to hit the ground*

$$s(t) = -16t^2 + 30$$
$$0 = -16t^2 + 30$$
$$-30 = -16t^2$$
$$t^2 = 1.875$$
$$t = 1.369s$$

Question 15

Multiple choice:

1) $\lim_{x\to\infty}\frac{4x+5}{8x-3}$ =

A) 1
B) 0.5
C) 3
D) 0.25

2) $\lim_{x\to\infty}\frac{6x^2-x}{3x^3+1}$

A) 3
B) 2
C) 0
D) 5

3) The derivative of $f(x) = \sqrt[14]{x^9}$ is ....

A) $f'(x) = \frac{9}{14}x^{\frac{5}{14}}$
B) $f'(x) = \frac{9}{14}x^{\frac{-5}{14}}$
C) $f'(x) = \frac{9}{14}x^{\frac{14}{5}}$
D) $f'(x) = -\frac{9}{14}x^{\frac{-5}{14}}$

4) The derivative of $f(x) = \frac{1}{x^5}$ is ....

A) $f'(x) = \frac{-5}{x^6}$

B) $f'(x) = \frac{-6}{x^6}$

C) $f'(x) = -5\sqrt{x^6}$

D) $f'(x) = 5x^4$

5) Value of $\int_0^6 (x+2)dx$ *is* ... ...

A) 48
B) 32
C) 30
D) 45

6) The antiderivative of $f(x) = \frac{2}{x^4}$ is ....

A) $\frac{2}{3x^3} + c$
B) $-\frac{2}{3x^3} + c$
C) $-\frac{3x^3}{2} + c$
D) $2x^{-4}$

Chapter 11 (11.1, 11.2, 11.3, 11.4 and 11.6)

Question 1

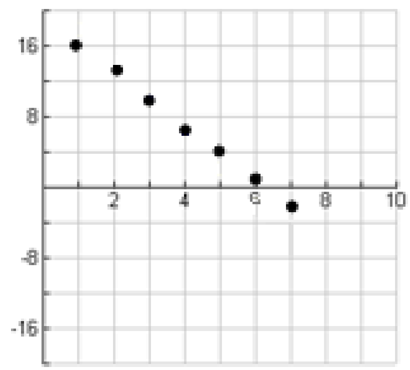
Consider the arithmetic sequence 16, 13, 10, ….

a. Find the next four terms of the sequence.

The common difference (d) = 13 − 16 = -3
The next four terms are 7, 4, 1, and -2.

b. Graph the first seven terms of the sequence.

The domain contains the terms {1, 2, 3, 4, 5, 6, 7} and the range contains the terms {16, 13, 10, 7, 4, 1, -2}. So, graph the corresponding ordered pairs.



Question 2

Find the indicated term of each arithmetic or geometric sequence.

a) $a_1 = 3, d = 7, n = 14$

$$\boldsymbol{a_n = a_1 + (n-1)d}$$

$$\boldsymbol{a_{14} = 3 + (14-1)(7) = 94}$$

b) $a_1 = 3, r = 2, n = 9$

$$\boldsymbol{a_n = a_1 r^{n-1}}$$

$$\boldsymbol{a_9 = 3(2)^{9-1} = 768}$$

Question 3

Write the equation for the *n*th term of the following sequences

a) Arithmetic sequence -4, 1, 6, 11, …….

$$\boldsymbol{d = 1 - (-4) = 5}$$
$$\boldsymbol{a_n = a_1 + (n-1)d}$$

$$\boldsymbol{a_n = -4 + (n-1)(5)}$$

$$\boldsymbol{a_n = -4 + 5n - 5}$$

$$\boldsymbol{a_n \ = -9 + 5n}$$

Question 4

Find **two geometric means** between 1 and 27

$$\boldsymbol{1, \dots, \dots, 27}$$

$$\boldsymbol{a_n = a_1 r^{n-1}}$$

$$\boldsymbol{27 = 1(r)^{4-1}}$$

$$\boldsymbol{27 = r^3}$$

$r = 3$

$1, 3, 9, 27$

Question 5

Find **four arithmetic means** between -8 and 22

$-8, \ldots, \ldots, \ldots, \ldots 22$

$a_n = a_1 + (n-1)d$

$22 = -8 + (6-1)d$

$22 = -8 + 5d$

$22 + 8 = 5d$

$30 = 5d$

$d = 6$

$-8, -2, 4, 10, 16$

Question 6

Find the sum of each series

a) Geometric series $\sum_{n=1}^{5} 2(4^{n-1})$

$\boldsymbol{a_1} = 2(4^{1-1}) = \mathbf{2}$
$\boldsymbol{n = 5}$
$\boldsymbol{r = 4}$

$$\boldsymbol{S_n = a_1(\frac{1-r^n}{1-r})}$$

$$\boldsymbol{S_n = 2\left(\frac{1-4^5}{1-4}\right) = 682}$$

b) Arithmetic series $\sum_{n=1}^{6}(2n+11)$

$\boldsymbol{a_1 = 2(1) + 11 = 13}$
$\boldsymbol{a_n = 2(6) + 11 = 23}$
$\boldsymbol{n = 6}$

$$\boldsymbol{S_n = n(\frac{a_1 + a_n}{2})}$$

$$\boldsymbol{S_n = 6\left(\frac{13+23}{2}\right) = 108}$$

Question 7

Find the sum of each **infinite series** if it exists

$$a)\ 14 + \frac{98}{3} + \frac{686}{9} + \cdots$$

$$r = \frac{(\frac{98}{3})}{14} = \frac{7}{3} > 1$$

Sum does not exist

$$b) \sum_{n=1}^{\infty} 2(\frac{2}{5})^{n}$$

$$a_1 = 2(\frac{2}{5})^1 = \frac{4}{5}$$

$$r = \frac{2}{5} < 1$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{4}{5}}{1-\frac{2}{5}} = \frac{4}{3}$$

Question 8

a) Find the sum of the first 50 positive integers

$$1 + 2 + 3 + 4 + \cdots 50$$

$$a_1 = 1$$

$d = 1$

$n = 50$

$S_n = n(\frac{a_1 + a_n}{2})$

$S_{50} = 50\left(\frac{1+50}{2}\right) = 1275$

b) Find the sum of the first 100 **<u>even natural</u>** numbers .

a_1 **= 2 , d = 2 , n= 100**

$S_n = \frac{n}{2}[2a_1 + d(n-1)]$

= $\frac{100}{2}[2(2) + 2(100-1)]$

= 50 (202) = 10100

<u>Question 9</u>

a) Find the first three terms of the following arithmetic series

$a_1 = 17 , a_n = 197 , S_n = 2247$

$S_n = n(\frac{a_1 + a_n}{2})$

$$2247 = n(\frac{17 + 197}{2})$$

$$2247 = n(\frac{214}{2})$$

$$2247 = n(107)$$

$$n = 21$$

$$a_n = a_1 + (n - 1)d$$

$$197 = 17 + (21 - 1)d$$

$$197 = 17 + 20d$$

$$180 = 20d$$

$$d = 9$$

$$17\,,26\,,35$$

b) Write an equation for the *n*th term of the following geometric sequence

a_4 = 12 and r = 4

Find a_1.

$a_n = a_1 r^{n-1}$

$12 = a_1(4^{4-1})$ $a_n = 12$, $r = 4$, and $n = 4$

$12 = a_1(64)$.

$\frac{12}{64} = a_1$

$\frac{3}{16} = a_1$

Write the equation.

$a_n = a_1 r^{n-1}$ $\quad\quad$ $\frac{3}{16} = a_1$ and r = 4

$a_n = \frac{3}{16} (4)^{n-1}$

c) Write $0.3\overline{21}$ as a fraction.

Ans :

$0.3\overline{21} = 0.3 + 0.021 + 0.00021 + \ldots\ldots\ldots\ldots\ldots\ldots$

$= 0.3 + \frac{a_1}{1-r}$ $\quad$ ....$a_1 = 0.021$, $r = \frac{0.00021}{0.21} = 0.01$

$= 0.3 + \frac{0.021}{1-0.01} = \frac{53}{165}$

Question 10 (Word Problems)

1) In a Physics experiment a steel ball on a flat track is accelerated and then allowed to roll freely . After the first minute , the ball has rolled 120 feet . Each minute the ball travels only 40% as far as it did during the preceding minute . How far does the ball travel?

$\boldsymbol{a_1 = 120}$

$\boldsymbol{r = 0.4} < 1$

$\boldsymbol{S = \frac{a_1}{1-r}}$

$S = \frac{120}{1-0.4} = 200\ feet$

2) Suppose you go to work for a company that pays \$0.01 on the first day, \$0.02 on the second day, \$0.04 on the third day and so on. If the daily wage keeps doubling, what will you total income be for working 31 days?

$a_1 = 0.01$
$a_2 = 0.02$
$a_3 = 0.04$
$r = 2$
$n = 31$

$S_n = a_1(\frac{1-r^n}{1-r})$

$S_{31} = 0.01\left(\frac{1-2^{31}}{1-2}\right) = \$21474836.47$

3) An auditorium has 20 seats on the first row, 24 seats on the second row, 28 seats on the third row, and so on and has 30 rows of seats. How many seats are in the theatre?
$a_1 = 20$
$a_2 = 24$
$a_3 = 28$

$d = 4$
$n = 30$

$a_n = a_1 + (n - 1)d$

$a_{30} = 20 + (30 - 1)(4) = 136$

$S_n = n(\frac{a_1 + a_n}{2})$

$S_{30} = 30\left(\frac{20 + 136}{2}\right) = 2340$

4) A ball is dropped from a height of 16 feet. Each time it drops, it rebounds 80% of the height from which it is falling. Find the total distance traveled in 15 bounces.

$a_1 = 16$
$r = 0.8$
$n = 15$

$S_n = a_1(\frac{1 - r^n}{1 - r})$

$S_{31} = 16\left(\frac{1 - 0.8^{15}}{1 - 0.8}\right) = 77.2\ feet$

5) Heavy rain in Lagos caused the river to rise . The river rose three inches the first day and each day after rose twice as much as the previous day . How much did the river rise in five days ?

$a_1 = 3$, $r = 2$, $n = 5$

$S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{3-3(2)^5}{1-2} = 93$

Question 11

Circle the correct answer

1) The sequence 1, 8 , 27 , 64,…. Is
a) Arithmetic sequence
b) Geometric sequence
c) Arithmetic and Geometric sequence
d) Neither Arithmetic nor Geometric

2) The sequence defined by 10 , 2 , - 6 ,… is
a) A geometric sequence with r= $\frac{1}{5}$
b) An arithmetic sequence with d=12
c) An arithmetic sequence with d=-8
d) A geometric sequence with r=-3

3) The integer -1590 is equivalent to

a) $\sum_{k=5}^{20}(-6-7k)$

b) $\sum_{k=1}^{20}(-6-7k)$

c) $\sum_{k=1}^{15}(-6-7k)$

d) $\sum_{k=1}^{20}(-7k+6)$

4) What is the sum of an infinite geometric series with the first term of 27 and a common ratio of $\frac{2}{3}$?

A) 81

B) 34

C) 65

D) 18

5) $\sum_{k=1}^{\infty}\frac{8}{3}\cdot(\frac{5}{6})^{k-1}$ = ...

A) 61

B) 72

C) 16

D) 100

6) The third term of $(x+2y)^7$ is ....

A) $48x^5y^2$

B) $84x^5\ y^2$
C) $84x^2\ y^5$
D) $48x^2y^5$

7) The coefficient of the seventh term of $(2a\ -2b)^8$ is ……
A) 7186
B) 7816
C) 7168
D) 7002

Question 12

a) Expand the following binomial $(2x + y)^5$

$$(2x+y)^5 = \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4(y)^1 + \binom{5}{2}(2x)^3(y)^2 + \binom{5}{3}(2x)^2(y)^3 + \binom{5}{4}(2x)^1(y)^4 + \binom{5}{5}(2x)^0(y)^5$$

$$(2x+y)^5 = 32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10x^1y^4 + y^5$$

b) Find the fifth term of $(5\text{x} - 4\text{a})^7$

$$\binom{7}{5}(5x)^2(-4a)^5 =$$

c) Find the probability of hiring 6 men and 2 women by expanding $(\text{x} + \text{y})^8$

$$(x+y)^8 = \binom{8}{0}(x)^8 + \binom{8}{1}(x)^7(y)^1 + \binom{8}{2}(x)^6(y)^2 + \binom{8}{3}(x)^5(y)^3 + \binom{8}{4}(x)^4(y)^4 + \binom{8}{5}(x)^3(y)^5 + \binom{8}{6}(x)^2(y)^6$$
$$5$$
$$+ \binom{8}{7}(x)^1(y)^7 + \binom{8}{8}(x)^0(y)^8$$

$$(x+y)^8 = x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$$

By adding the coefficients of the polynomial, there are 256 combinations of the males and females

$28x^6y^2$ **Represents the number of combination with 6 males and 2 females,** $P = \frac{28}{256}$