1. (a) At (x_1, y_1), slope $= 0$.

At (x_2, y_2), slope $= \frac{5}{2}$.

(b) At (x_1, y_1), slope $= -\frac{5}{2}$.

At (x_2, y_2), slope $= 2$.

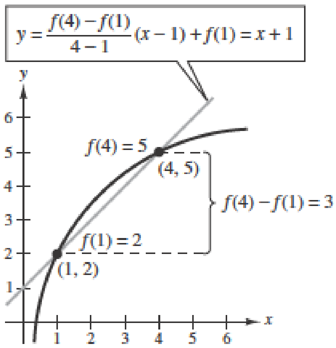
2. (a) At (x_1, y_1), slope $= \frac{2}{3}$.

At (x_2, y_2), slope $= -\frac{2}{5}$.

(b) At (x_1, y_1), slope $= \frac{5}{4}$.

At (x_2, y_2), slope $= \frac{5}{4}$.

3. (a), (b)



(c) $$y = \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$$
$$= \frac{3}{3}(x - 1) + 2$$
$$= 1(x - 1) + 2$$
$$= x + 1$$

4. (a) $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{5 - 2}{3} = 1$

$\dfrac{f(4) - f(3)}{4 - 3} \approx \dfrac{5 - 4.75}{1} = 0.25$

So, $\dfrac{f(4) - f(1)}{4 - 1} > \dfrac{f(4) - f(3)}{4 - 3}$.

(b) The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. So,

$\dfrac{f(4) - f(1)}{4 - 1} < f'(1)$.

5. $g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

6. $f(x) = 3 - 5x$ is a line. Slope $= -5$

7. Slope at $(1, 5)$

$$\begin{aligned} &= \lim_{\Delta x \to 0} \frac{g(1 + \Delta x) - g(1)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{6 - (1 + \Delta x)^2 - 5}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{6 - 1 - 2(\Delta x) - (\Delta x)^2 - 5}{\Delta x} \\ &= \lim_{\Delta x \to 0} (-2 - \Delta x) = -2 \end{aligned}$$

8. Slope at $(2, -5)$

$$\begin{aligned} &= \lim_{\Delta x \to 0} \frac{g(2 + \Delta x) - g(2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(2 + \Delta x)^2 - 9 - (-5)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4 + 4(\Delta x) + (\Delta x)^2 - 4}{\Delta x} \\ &= \lim_{\Delta x \to 0} (4 + \Delta x) = 4 \end{aligned}$$

9. Slope at $(-2, 7)$
$$= \lim_{\Delta t \to 0} \frac{h(-2 + \Delta t) - h(-2)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{(-2 + \Delta t)^2 + 3 - 7}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{4 - 4(\Delta t) + (\Delta t)^2 - 4}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (-4 + \Delta t) = -4$$

10. Slope at $(0, 0)$
$$= \lim_{\Delta t \to 0} \frac{f(0 + \Delta t) - f(0)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (3 - \Delta t) = 3$$

11. $f(x) = 7$
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{7 - 7}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 0 = 0$$

12. $g(x) = -3$
$$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-3 - (-3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

13. $f(x) = 3x + 2$
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[3(x + \Delta x) + 2] - [3x + 2]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 3 = 3$$

14. $f(x) = -10x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-10(x + \Delta x) - (-10x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-10\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (-10) = -10$$

15. $f(x) = 8 - \frac{1}{5}x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8 - \frac{1}{5}(x + \Delta x) - \left(8 - \frac{1}{5}x\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{5}(\Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(-\frac{1}{5}\right) = -\frac{1}{5}$$

16. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

17. $f(x) = 2 - x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2 - (x + \Delta x)^2 - (2 - x^2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2 - x^2 - 2x(\Delta x) - (\Delta x)^2 - 2 + x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-2x(\Delta x) - (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} [-2x - \Delta x] = -2x$$

18. $f(x) = x^2 + x - 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 1) = 2x + 1$$

19. $f(x) = x^3 + x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + (x + \Delta x)^2\right] - \left[x^3 + x^2\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + x^2 + 2x\,\Delta x + (\Delta x)^2 - x^3 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x\,\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 + 2x + (\Delta x)\right) = 3x^2 + 2x$$

20. $f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12\right) = 3x^2 - 12$$

21. $f(x) = \dfrac{1}{x - 1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 1} - \dfrac{1}{x - 1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x - 1) - (x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 1)(x - 1)}$$

$$= -\frac{1}{(x - 1)^2}$$

22. $f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x + \Delta x)^2} - \dfrac{1}{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{\Delta x(x + \Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x(x + \Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x + \Delta x)^2 x^2}$$

$$= \frac{-2x}{x^4}$$

$$= -\frac{2}{x^3}$$

23. $f(x) = \sqrt{x + 4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 4} - \sqrt{x + 4}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} \right)$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 4) - (x + 4)}{\Delta x\left[\sqrt{x + \Delta x + 4} + \sqrt{x + 4}\right]}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} = \frac{1}{\sqrt{x + 4} + \sqrt{x + 4}} = \frac{1}{2\sqrt{x + 4}}$$

24. $f(x) = \dfrac{4}{\sqrt{x}}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{4}{\sqrt{x + \Delta x}} - \dfrac{4}{\sqrt{x}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4\sqrt{x} - 4\sqrt{x + \Delta x}}{\Delta x\sqrt{x}\sqrt{x + \Delta x}} \cdot \left(\frac{\sqrt{x} + \sqrt{x + \Delta x}}{\sqrt{x} + \sqrt{x + \Delta x}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{4x - 4(x + \Delta x)}{\Delta x\sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{-4}{\sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \frac{-4}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-2}{x\sqrt{x}}$$

25. (a) $f(x) = x^2 + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 3\right] - \left[x^2 + 3\right]}{\Delta x}$$

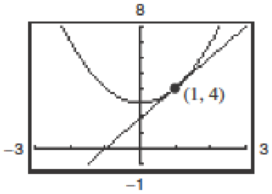
$$= \lim_{\Delta x \to 0} \frac{2x\,\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $(1, 4)$, the slope of the tangent line is $m = 2(1) = 2$. The equation of the tangent line is

$$y - 4 = 2(x - 1)$$
$$y - 4 = 2x - 2$$
$$y = 2x + 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $(1, 4)$.

26. (a) $f(x) = x^2 + 3x + 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 3(x + \Delta x) + 4 - (x^2 + 3x + 4)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + 3x + 3\Delta x + 4 - x^2 - 3x - 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + 3\Delta x}{\Delta x}$$

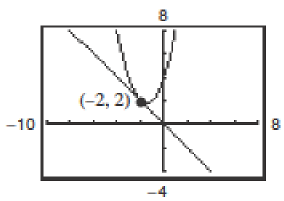
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 3) = 2x + 3$$

At $(-2, 2)$, the slope of the tangent line is $m = 2(-2) + 3 = -1$. The equation of the tangent line is

$$y - 2 = -1(x + 2)$$

$$y = -x$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = -1$ at $(-2, 2)$.

27. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

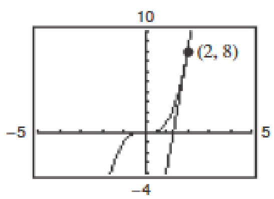
$$= \lim_{\Delta x \to 0} \frac{3x^2 \Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$. The equation of the tangent line is

$$y - 8 = 12(x - 2)$$
$$y = 12x - 16.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 12$ at $(2, 8)$.

28. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + 1\right] - (x^3 + 1)}{\Delta x}$$

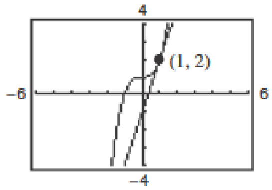
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] = 3x^2$$

At $(1, 2)$, the slope of the tangent is $m = 3(1)^2 = 3$. The equation of the tangent line is

$$y - 2 = 3(x - 1)$$
$$y = 3x - 1.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 3$ at $(1, 2)$.

29. (a) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

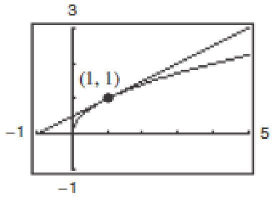
At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

The equation of the tangent line is

$$y - 1 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x + \frac{1}{2}.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{1}{2}$ at $(1, 1)$.

30. (a) $f(x) = \sqrt{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x-1} + \sqrt{x-1}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

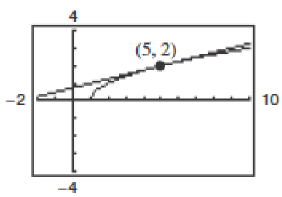
At $(5, 2)$, the slope of the tangent line is $m = \dfrac{1}{2\sqrt{5-1}} = \dfrac{1}{4}$

The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x-5)$$

$$y = \frac{1}{4}x + \frac{3}{4}$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{1}{4}$ at $(5, 2)$.

31. (a) $f(x) = \dfrac{1}{x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x+1} - \dfrac{1}{x+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+1) - (x+\Delta x+1)}{\Delta x(x+\Delta x+1)(x+1)}$$

$$= \lim_{\Delta x \to 0} -\frac{1}{(x+\Delta x+1)(x+1)}$$

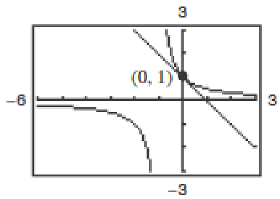
$$= -\frac{1}{(x+1)^2}$$

At $(0, 1)$, the slope of the tangent line is

$$m = \frac{-1}{(0+1)^2} = -1.$$

The equation of the tangent line is $y = -x + 1$.

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -1$ at $(0, 1)$.

32. (a) $f(x) = x + \dfrac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) + \dfrac{4}{x + \Delta x} - \left(x + \dfrac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x + \Delta x)(x + \Delta x) + 4x - x^2(x + \Delta x) - 4(x + \Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x + \Delta x)}$$

$$= \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

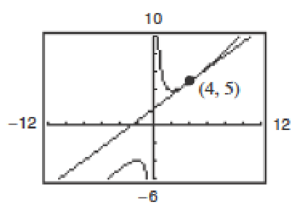
At $(4, 5)$, the slope of the tangent line is $m = 1 - \dfrac{4}{16} = \dfrac{3}{4}$.

The equation of the tangent line is

$$y - 5 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x + 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{3}{4}$ at $(4, 5)$.

33. Using the limit definition of derivative, $f'(x) = 2x$. Because the slope of the given line is 2, you have

$$2x = 2$$
$$x = 1$$

At the point $(1, 1)$ the tangent line is parallel to $2x - y + 1 = 0$. The equation of this line is

$$y - 1 = 2(x - 1)$$
$$y = 2x - 1.$$

34. Using the limit definition of derivative, $f'(x) = 4x$. Because the slope of the given line is -4, you have

$$4x = -4$$
$$x = -1.$$

At the point $(-1, 2)$ the tangent line is parallel to $4x + y + 3 = 0$. The equation of this line is

$$y - 2 = -4(x + 1)$$
$$y = -4x - 2.$$

35. From Exercise 27 we know that $f'(x) = 3x^2$. Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x = \pm 1.$$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$. These lines have equations

$$y - 1 = 3(x - 1) \quad \text{and} \quad y + 1 = 3(x + 1)$$
$$y = 3x - 2 \qquad\qquad y = 3x + 2.$$

36. Using the limit definition of derivative, $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x^2 = 1 \Rightarrow x = \pm 1.$$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$$y - 3 = 3(x - 1) \quad \text{and} \quad y - 1 = 3(x + 1)$$
$$y = 3x \qquad\qquad\qquad y = 3x + 4.$$

37. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2x\sqrt{x}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2}$$
$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$
$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}.$$

38. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x-1)^{3/2}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$

$$1 = (x-1)^{3/2}$$

$$1 = x - 1 \Rightarrow x = 2.$$

At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 2.$$

39. $f(x) = x \Rightarrow f'(x) = 1$ Matches (b).

40. $f(x) = x^2 \Rightarrow f'(x) = 2x$ Matches (d).

41. $f(x) = \sqrt{x} \Rightarrow f'(x)$ Matches (a).

(decreasing slope as $x \to \infty$)

42. f' does not exist at $x = 0$. Matches (c).

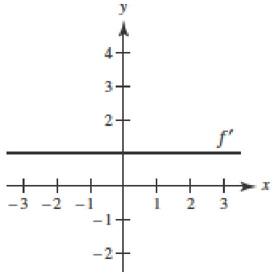
43. $g(4) = 5$ because the tangent line passes through $(4, 5)$.

$$g'(4) = \frac{5-0}{4-7} = -\frac{5}{3}$$

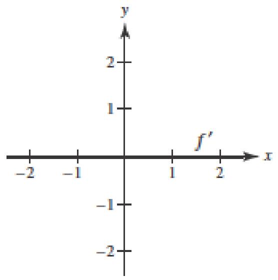
44. $h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

$$h'(-1) = \frac{6-4}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

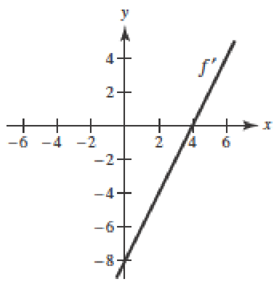
45. The slope of the graph of f is 1 for all x-values.



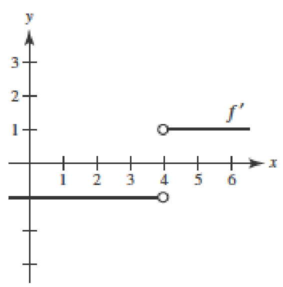
46. The slope of the graph of f is 0 for all x-values.



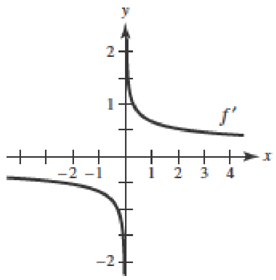
47. The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



48. The slope of the graph of f is –1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.



49. The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.



50. The slope is positive for $-2 < x < 0$ and negative for $0 < x < 2$. The slope is undefined at $x = \pm 2$, and 0 at $x = 0$.