51. $\lim_{x\to 0} \dfrac{(x+2)}{\cot x} = \lim_{x\to 0}\left[(x+2)\tan x\right] = 0$

52. $\lim_{x\to \pi} \dfrac{\sqrt{x}}{\csc x} = \lim_{x\to \pi}\left(\sqrt{x}\sin x\right) = 0$

53. $\lim_{x\to (1/2)^-} x^2 \tan \pi x = \infty$ and

$\lim_{x\to (1/2)^+} x^2 \tan \pi x = -\infty$. Therefore,

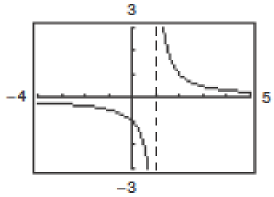
$\lim_{x\to (1/2)} x^2 \tan \pi x$ does not exist.

54. $\lim_{x\to (1/2)^-} x \sec(\pi x) = \infty$ and $\lim_{x\to (1/2)^+} x \sec(\pi x) = -\infty$.

Therefore, $\lim_{x\to (1/2)} x \sec(\pi x)$ does not exist.

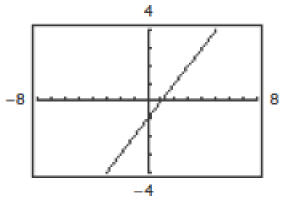
55. $f(x) = \dfrac{x^2+x+1}{x^3-1} = \dfrac{x^2+x+1}{(x-1)(x^2+x+1)}$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \dfrac{1}{x-1} = \infty$



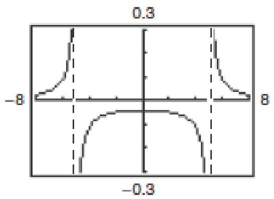
56. $f(x) = \dfrac{x^3-1}{x^2+x+1} = \dfrac{(x-1)(x^2+x+1)}{x^2+x+1}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x-1) = 0$



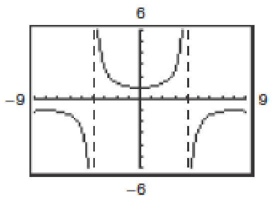
57. $f(x) = \dfrac{1}{x^2 - 25}$

$\lim\limits_{x \to 5^-} f(x) = -\infty$



58. $f(x) = \sec \dfrac{\pi x}{8}$

$\lim\limits_{x \to 4^+} f(x) = -\infty$



59. A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an infinite limit. ∞ is not a number. Rather, the symbol

$\lim\limits_{x \to c} f(x) = \infty$

says how the limit fails to exist.

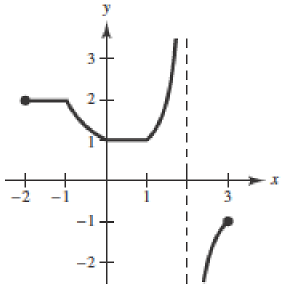
60. The line $x = c$ is a vertical asymptote if the graph of f approaches $\pm\infty$ as x approaches c.

61. One answer is

$$f(x) = \frac{x - 3}{(x - 6)(x + 2)} = \frac{x - 3}{x^2 - 4x - 12}.$$

62. No. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

63.



64. No, it is not true. Consider $p(x) = x^2 - 1$. The function

$$f(x) = \frac{x^2 - 1}{x - 1} = \frac{p(x)}{x - 1}$$

has a hole at $(1, 2)$, not a vertical asymptote.

65. $m = \dfrac{m_0}{\sqrt{1 - (v^2/c^2)}}$

$$\lim_{v \to c^-} m = \lim_{v \to c^-} \frac{m_0}{\sqrt{1 - (v^2/c^2)}} = \infty$$

66. $P = \dfrac{k}{V}$

$$\lim_{V \to 0^+} \frac{k}{V} = k(\infty) = \infty$$

(In this case you know that $k > 0$.)

67. (a) $r = 50\pi \sec^2 \dfrac{\pi}{6} = \dfrac{200\pi}{3}$ ft/sec

(b) $r = 50\pi \sec^2 \dfrac{\pi}{3} = 200\pi$ ft/sec

(c) $\displaystyle\lim_{\theta \to (\pi/2)^-} \left[50\pi \sec^2 \theta\right] = \infty$

68. (a) $r = \dfrac{2(7)}{\sqrt{625 - 49}} = \dfrac{7}{12}$ ft/sec

(b) $r = \dfrac{2(15)}{\sqrt{625 - 225}} = \dfrac{3}{2}$ ft/sec

(c) $\displaystyle\lim_{x \to 25^-} \frac{2x}{\sqrt{625 - x^2}} = \infty$

69. (a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$50 = \frac{2d}{(d/x) + (d/y)}$$

$$50 = \frac{2xy}{y + x}$$

$$50y + 50x = 2xy$$

$$50x = 2xy - 50y$$

$$50x = 2y(x - 25)$$

$$\frac{25x}{x - 25} = y$$

Domain: $x > 25$

(b)

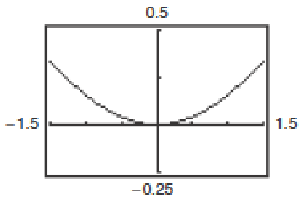
x	30	40	50	60
y	150	66.667	50	42.857

(c) $\displaystyle\lim_{x \to 25^+} \frac{25x}{\sqrt{x - 25}} = \infty$

As x gets close to 25 mi/h, y becomes larger and larger.

70. (a)

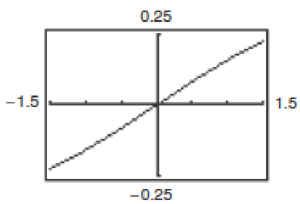
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0411	0.0067	0.0017	≈ 0	≈ 0	≈ 0



$$\lim_{x\to 0^+} \frac{x - \sin x}{x} = 0$$

(b)

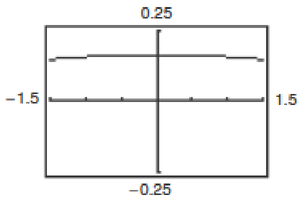
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0823	0.0333	0.0167	0.0017	≈ 0	≈ 0



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^2} = 0$$

(c)

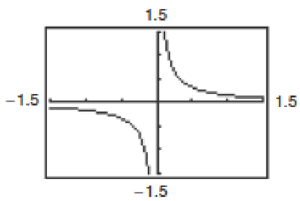
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.1646	0.1663	0.1666	0.1667	0.1667	0.1667



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^3} = 0.1667\ (1/6)$$

(d)

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.3292	0.8317	1.6658	16.67	166.7	1667.0



$$\lim_{x\to 0^+} \frac{x - \sin x}{x^4} = \infty$$

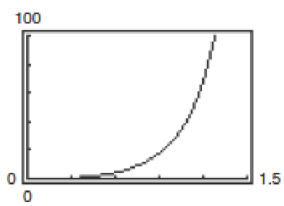
For $n > 3$, $\displaystyle\lim_{x\to 0^+} \frac{x - \sin x}{x^n} = \infty$.

71. (a) $A = \frac{1}{2}bh - \frac{1}{2}r^2\theta = \frac{1}{2}(10)(10\tan\theta) - \frac{1}{2}(10)^2\theta = 50\tan\theta - 50\,\theta$

Domain: $\left(0, \frac{\pi}{2}\right)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\displaystyle\lim_{\theta\to\pi/2^-} A = \infty$

72. (a) Because the circumference of the motor is half that of the saw arbor, the saw makes $1700/2 = 850$ revolutions per minute.

(b) The direction of rotation is reversed.

(c) $2(20 \cot \phi) + 2(10 \cot \phi)$: straight sections. The angle subtended in each circle is $2\pi - \left(2\left(\frac{\pi}{2} - \phi\right)\right) = \pi + 2\phi$.

So, the length of the belt around the pulleys is $20(\pi + 2\phi) + 10(\pi + 2\phi) = 30(\pi + 2\phi)$.

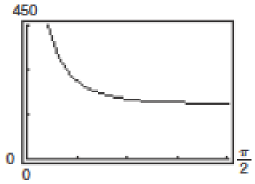
Total length $= 60 \cot \phi + 30(\pi + 2\phi)$

Domain: $\left(0, \frac{\pi}{2}\right)$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e)



(f) $\lim_{\phi \to (\pi/2)^-} L = 60\pi \approx 188.5$

(All the belts are around pulleys.)

(g) $\lim_{\phi \to 0^+} L = \infty$

73. False. For instance, let

$$f(x) = \frac{x^2 - 1}{x - 1} \text{ or}$$

$$g(x) = \frac{x}{x^2 + 1}.$$

74. True

75. False. The graphs of $y = \tan x, y = \cot x, y = \sec x$ and $y = \csc x$ have vertical asymptotes.

76. False. Let

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 3, & x = 0. \end{cases}$$

The graph of f has a vertical asymptote at $x = 0$, but $f(0) = 3$.

77. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{x^4}$, and $c = 0$.

$\lim_{x\to 0} \dfrac{1}{x^2} = \infty$ and $\lim_{x\to 0} \dfrac{1}{x^4} = \infty$, but

$$\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x^4}\right) = \lim_{x\to 0}\left(\frac{x^2 - 1}{x^4}\right) = -\infty \neq 0.$$

78. Given $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$:

(2) Product:

If $L > 0$, then for $\varepsilon = L/2 > 0$ there exists $\delta_1 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_1$. So, $L/2 < g(x) < 3L/2$. Because $\lim_{x\to c} f(x) = \infty$ then for $M > 0$, there exists $\delta_2 > 0$ such that $f(x) > M(2/L)$ whenever $|x - c| < \delta_2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have $f(x)g(x) > M(2/L)(L/2) = M$. Therefore $\lim_{x\to c} f(x)g(x) = \infty$. The proof is similar for $L < 0$.

(3) Quotient: Let $\varepsilon > 0$ be given.

There exists $\delta_1 > 0$ such that $f(x) > 3L/2\varepsilon$ whenever $0 < |x - c| < \delta_1$ and there exists $\delta_2 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_2$. This inequality gives us $L/2 < g(x) < 3L/2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have

$$\left|\frac{g(x)}{f(x)}\right| < \frac{3L/2}{3L/2\varepsilon} = \varepsilon.$$

Therefore, $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$.

79. Given $\lim_{x\to c} f(x) = \infty$, let $g(x) = 1$. Then

$\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$ by Theorem 1.15.

80. Given $\lim_{x\to c}\frac{1}{f(x)} = 0$. Suppose $\lim_{x\to c} f(x)$ exists and equals L.

Then, $\lim_{x\to c}\frac{1}{f(x)} = \frac{\lim_{x\to c} 1}{\lim_{x\to c} f(x)} = \frac{1}{L} = 0.$

This is not possible. So, $\lim_{x\to c} f(x)$ does not exist.

81. $f(x) = \frac{1}{x-3}$ is defined for all $x > 3$. Let $M > 0$ be given. You need $\delta > 0$ such that

$f(x) = \frac{1}{x-3} > M$ whenever $3 < x < 3 + \delta$.

Equivalently, $x - 3 < \frac{1}{M}$ whenever

$|x-3| < \delta, x > 3$.

So take $\delta = \frac{1}{M}$. Then for $x > 3$ and

$|x-3| < \delta, \frac{1}{x-3} > \frac{1}{8} = M$ and so $f(x) > M$.

82. $f(x) = \frac{1}{x-5}$ is defined for all $x < 5$. Let $N < 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x-5} < N$ whenever $5 - \delta < x < 5$. Equivalently, $x - 5 > \frac{1}{N}$ whenever $|x-5| < \delta, x < 5$. Equivalently, $\frac{1}{|x-5|} < -\frac{1}{N}$ whenever $|x-5| < \delta, x < 5$. So take $\delta = -\frac{1}{N}$. Note that $\delta > 0$ because $N < 0$. For $|x-5| < \delta$ and $x < 5, \frac{1}{|x-5|} > \frac{1}{\delta} = -N$, and $\frac{1}{x-5} = -\frac{1}{|x-5|} < N$.