1. $f(x) = \dfrac{1}{x - 4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$$\lim_{x \to 4^-} f(x) = -\infty$$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$$\lim_{x \to 4^+} f(x) = \infty$$

2. $f(x) = \dfrac{-1}{x - 4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$$\lim_{x \to 4^-} f(x) = \infty.$$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$$\lim_{x \to 4^+} f(x) = -\infty.$$

3. $f(x) = \dfrac{1}{(x - 4)^2}$

As x approaches 4 from the left or right, $(x - 4)^2$ is a small positive number. So,

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^-} f(x) = \infty.$$

4. $f(x) = \dfrac{-1}{(x - 4)^2}$

As x approaches 4 from the left or right, $(x - 4)^2$ is a small positive number. So,

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = -\infty.$$

5. $\displaystyle\lim_{x \to -2^+} 2\left|\frac{x}{x^2 - 4}\right| = \infty$

$$\lim_{x \to -2^-} 2\left|\frac{x}{x^2 - 4}\right| = \infty$$

6. $\lim_{x\to -2^+} \frac{1}{x+2} = \infty$

$\lim_{x\to -2^-} \frac{1}{x+2} = -\infty$

7. $\lim_{x\to -2^+} \tan\frac{\pi x}{4} = -\infty$

$\lim_{x\to -2^-} \tan\frac{\pi x}{4} = \infty$

8. $\lim_{x\to -2^+} \sec\frac{\pi x}{4} = \infty$

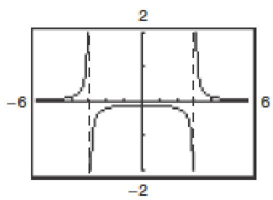
$\lim_{x\to -2^-} \sec\frac{\pi x}{4} = -\infty$

9. $f(x) = \frac{1}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	0.308	1.639	16.64	166.6	–166.7	–16.69	–1.695	–0.364

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to -3^+} f(x) = -\infty$

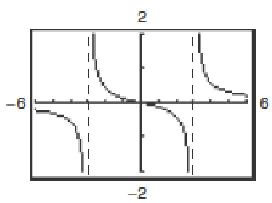


10. $f(x) = \frac{x}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.077	–5.082	–50.08	–500.1	499.9	49.92	4.915	0.9091

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

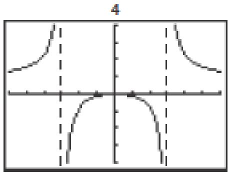


11. $f(x) = \dfrac{x^2}{x^2 - 9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	3.769	15.75	150.8	1501	-1499	-149.3	-14.25	-2.273

$$\lim_{x\to -3^-} f(x) = \infty$$

$$\lim_{x\to -3^+} f(x) = -\infty$$

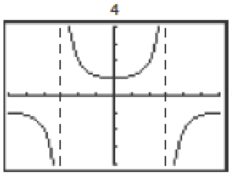


12. $f(x) = \sec\dfrac{\pi x}{6}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	-3.864	-19.11	-191.0	-1910	1910	191.0	19.11	3.864

$$\lim_{x\to -3^-} f(x) = -\infty$$

$$\lim_{x\to -3^+} f(x) = \infty$$



13. $\lim_{x\to 0^+} \dfrac{1}{x^2} = \infty = \lim_{x\to 0^-} \dfrac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

14. $\lim_{x\to 2^+} \dfrac{4}{(x-2)^3} = \infty$

$\lim_{x\to 2^-} \dfrac{4}{(x-2)^3} = -\infty$

Therefore, $x = 2$ is a vertical asymptote.

15. No vertical asymptote because the denominator is never zero.

16. $\lim_{x\to -2^-} \frac{x^2}{x^2-4} = \infty$ and $\lim_{x\to -2^+} \frac{x^2}{x^2-4} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x\to 2^-} \frac{x^2}{x^2-4} = -\infty$ and $\lim_{x\to 2^+} \frac{x^2}{x^2-4} = \infty$

Therefore, $x = 2$ is a vertical asymptote.

17. $\lim_{s\to -5^-} h(s) = -\infty$ and $\lim_{s\to -5^+} h(s) = \infty.$

Therefore, $s = -5$ is a vertical asymptote.

$\lim_{s\to 5^-} h(s) = -\infty$ and $\lim_{s\to 5^+} h(s) = \infty.$

Therefore, $s = 5$ is a vertical asymptote.

18. No vertical asymptote because the denominator is never zero.

19. $\lim_{x\to 0^-} \frac{2+x}{x^2(1-x)} = \lim_{x\to 0^+} \frac{2+x}{x^2(1-x)} = \infty$

Therefore, $x = 0$ is a vertical asymptote.

$\lim_{x\to 1^-} \frac{2+x}{x^2(1-x)} = \infty$

$\lim_{x\to 1^+} \frac{2+x}{x^2(1-x)} = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

20. $\lim_{x\to 2^+} \frac{x^2-2}{(x-2)(x+1)} = \infty$

$\lim_{x\to 2^-} \frac{x^2-2}{(x-2)(x+1)} = -\infty$

Therefore, $x = 2$ is a vertical asymptote.

$\lim_{x\to -1^+} \frac{x^2-2}{(x-2)(x+1)} = \infty$

$\lim_{x\to -1^-} \frac{x^2-2}{(x-2)(x+1)} = -\infty$

Therefore, $x = -1$ is a vertical asymptote.

21. $\lim_{t\to 0^+}\left(1-\frac{4}{t^2}\right) = -\infty = \lim_{t\to 0^-}\left(1-\frac{4}{t^2}\right)$

Therefore, $t = 0$ is a vertical asymptote.

22. $g(x) = \frac{(1/2)x^3 - x^2 - 4x}{3x^2 - 6x - 24} = \frac{1}{6}\frac{x(x^2 - 2x - 8)}{x^2 - 2x - 8}$

$= \frac{1}{6}x, \quad x \neq -2, 4$

No vertical asymptote. The graph has holes at $x = -2$ and $x = 4$.

23. $f(x) = \frac{3}{x^2 + x - 2} = \frac{3}{(x+2)(x-1)}$

Vertical asymptotes at $x = -2$ and $x = 1$.

24. $f(x) = \frac{4(x^2 + x - 6)}{x(x^3 - 2x^2 - 9x + 18)}$

$= \frac{4(x+3)(x-2)}{x(x-2)(x^2-9)}$

$= \frac{4}{x(x-3)}, x \neq -3, 2$

Vertical asymptotes at $x = 0$ and $x = 3$. The graph has holes at $x = -3$ and $x = 2$.

25. $h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2} = \frac{(x+2)(x-2)}{(x+2)(x^2+1)}$

has no vertical asymptote because

$\lim_{x\to -2} h(x) = \lim_{x\to -2}\frac{x-2}{x^2+1} = -\frac{4}{5}.$

The graph has a hole at $x = -2$.

26. $f(x) = \frac{x^3 + 1}{x + 1} = \frac{(x+1)(x^2 - x + 1)}{x + 1}$

has no vertical asymptote because

$\lim_{x\to -1} f(x) = \lim_{x\to -1}(x^2 - x + 1) = 3.$

The graph has a hole at $x = -1$.

27. $h(t) = \dfrac{t(t-2)}{(t-2)(t+2)(t^2+4)}$

$= \dfrac{t}{(t+2)(t^2+4)}, t \neq 2$

Vertical asymptote at $t = -2$. The graph has a hole at $t = 2$.

28. $f(x) = \dfrac{(x-5)(x+3)}{(x-5)(x^2+1)} = \dfrac{x+3}{x^2+1}, x \neq 5$

No vertical asymptote. The graph has a hole at $x = 5$.

29. $f(x) = \sec \pi x = \dfrac{1}{\cos \pi x}$ has vertical asymptotes at

$x = \dfrac{2n+1}{2}$, n any integer.

30. $f(x) = \tan \pi x = \dfrac{\sin \pi x}{\cos \pi x}$ has vertical asymptotes at

$x = \dfrac{2n+1}{2}$, n any integer.

31. $g(\theta) = \dfrac{\tan \theta}{\theta} = \dfrac{\sin \theta}{\theta \cos \theta}$ has vertical asymptotes at

$\theta = \dfrac{(2n+1)\pi}{2} = \dfrac{\pi}{2} + n\pi$, n any interger.

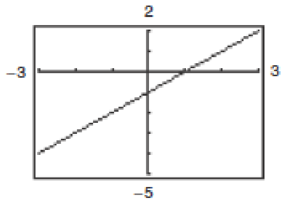
There is no vertical asymptote at $\theta = 0$ because

$\lim_{\theta \to 0} \dfrac{\tan \theta}{\theta} = 1.$

32. $s(t) = \dfrac{t}{\sin t}$ has vertical asymptotes at $t = n\pi$, n a nonzero integer. There is no vertical asymptote at $t = 0$ since

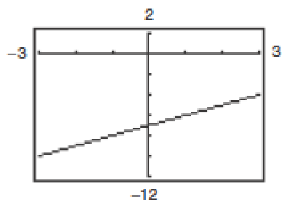
$\lim_{t \to 0} \dfrac{t}{\sin t} = 1.$

33. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = \lim_{x \to -1}(x - 1) = -2$



Removable discontinuity at $x = -1$

34. $\lim_{x \to -1} \frac{x^2 - 6x - 7}{x + 1} = \lim_{x \to -1}(x - 7) = -8$

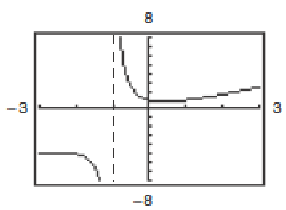


Removable discontinuity at $x = -1$

35. $\lim_{x \to -1^+} \frac{x^2 + 1}{x + 1} = \infty$

$\lim_{x \to -1^-} \frac{x^2 + 1}{x + 1} = -\infty$

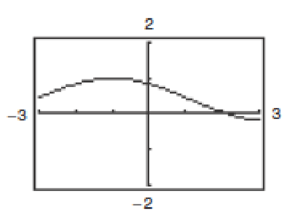
Vertical asymptote at $x = -1$



36. $\lim_{x \to -1} \frac{\sin(x + 1)}{x + 1} = 1$

Removable discontinuity at

$x = -1$



37. $\lim_{x\to -1^+} \frac{1}{x+1} = \infty$

38. $\lim_{x\to 1^-} \frac{-1}{(x-1)^2} = -\infty$

39. $\lim_{x\to 1^+} \frac{2+x}{1-x} = -\infty$

40. $\lim_{x\to 2^+} \frac{x}{x-2} = \infty$

41. $\lim_{x\to 4^-} \frac{x^2}{x^2+16} = \frac{1}{2}$

42. $\lim_{x\to 1^+} \frac{x^2}{(x-1)^2} = \infty$

43. $$\lim_{x\to -(1/2)^+} \frac{6x^2+x-1}{4x^2-4x-3} = \lim_{x\to -(1/2)^+} \frac{(3x-1)(2x+1)}{(2x-3)(2x+1)} = \lim_{x\to -(1/2)^+} \frac{3x-1}{2x-3} = \frac{5}{8}$$

44. $$\lim_{x\to -3^-} \frac{x+3}{(x^2+x-6)} = \lim_{x\to -3^-} \frac{x+3}{(x+3)(x-2)} = \lim_{x\to -3^-} \frac{1}{x-2} = -\frac{1}{5}$$

45. $\lim_{x\to 3} \frac{x-2}{x^2} = \frac{1}{9}$

46. $\lim_{x\to 1} \frac{x-1}{(x^2+1)(x-1)} = \lim_{x\to 1} \frac{1}{x^2+1} = \frac{1}{2}$

47. $\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right) = \infty$

48. $\lim_{x\to 0^-} \left(1 + \frac{1}{x}\right) = -\infty$

49. $\lim_{x\to (\pi/2)^+} \frac{-2}{\cos x} = \infty$

50. $\lim_{x \to 0^+} \frac{2}{\sin x} = \infty$