1. (a) $\lim_{x\to 4^+} f(x) = 3$

(b) $\lim_{x\to 4^-} f(x) = 3$

(c) $\lim_{x\to 4} f(x) = 3$

The function is continuous at $x = 4$ and is continuous on $(-\infty, \infty)$.

2. (a) $\lim_{x\to -2^+} f(x) = -2$

(b) $\lim_{x\to -2^-} f(x) = -2$

(c) $\lim_{x\to -2} f(x) = -2$

The function is continuous at $x = -2$.

3. (a) $\lim_{x\to 3^+} f(x) = 0$

(b) $\lim_{x\to 3^-} f(x) = 0$

(c) $\lim_{x\to 3} f(x) = 0$

The function is NOT continuous at $x = 3$.

4. (a) $\lim_{x\to -3^+} f(x) = 3$

(b) $\lim_{x\to -3^-} f(x) = 3$

(c) $\lim_{x\to -3} f(x) = 3$

The function is NOT continuous at $x = -3$ because $f(-3) = 4 \neq \lim_{x\to -3} f(x)$.

5. (a) $\lim_{x\to 2^+} f(x) = -3$

(b) $\lim_{x\to 2^-} f(x) = 3$

(c) $\lim_{x\to 2} f(x)$ does not exist

The function is NOT continuous at $x = 2$.

6. (a) $\lim_{x\to-1^+} f(x) = 0$

(b) $\lim_{x\to-1^-} f(x) = 2$

(c) $\lim_{x\to-1} f(x)$ does not exist.

The function is NOT continuous at $x = -1$.

7. $\lim_{x\to8^+} \frac{1}{x+8} = \frac{1}{8+8} = \frac{1}{16}$

8. $\lim_{x\to5^-} -\frac{3}{x+5} = -\frac{3}{5+5} = -\frac{3}{10}$

9. $\lim_{x\to5^+} \frac{x-5}{x^2-25} = \lim_{x\to5^+} \frac{1}{x+5} = \frac{1}{10}$

10. $\lim_{x\to2^+} \frac{2-x}{x^2-4} = \lim_{x\to2^+} -\frac{1}{x+2} = -\frac{1}{4}$

11. $\lim_{x\to9^-} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to9^-} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$

$$= \lim_{x\to9^-} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x\to9^-} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$$

12. $\lim_{x\to-3^-} \frac{x}{\sqrt{x^2-9}}$ does not exist because

$\frac{x}{\sqrt{x^2-9}}$ decreases without bound as $x \to -3^-$.

13. $\lim_{x\to0^-} \frac{|x|}{x} = \lim_{x\to0^-} \frac{-x}{x} = -1$

14. $\lim_{x\to10^+} \frac{|x-10|}{x-10} = \lim_{x\to10^+} \frac{x-10}{x-10} = 1$

15. $\lim_{\Delta x\to 0^+}\frac{(x+\Delta x)^2+(x+\Delta x)-(x^2+x)}{\Delta x} = \lim_{\Delta x\to 0^+}\frac{x^2+2x(\Delta x)+(\Delta x)^2+x+\Delta x-x^2-x}{\Delta x}$

$= \lim_{\Delta x\to 0^+}\frac{2x(\Delta x)+(\Delta x)^2+\Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0^+}(2x+\Delta x+1)$

$= 2x+0+1 = 2x+1$

16. $\lim_{\Delta x\to 0^-}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0^-}\frac{x-(x+\Delta x)}{x(x+\Delta x)}\cdot\frac{1}{\Delta x} = \lim_{\Delta x\to 0^-}\frac{-\Delta x}{x(x+\Delta x)}\cdot\frac{1}{\Delta x}$

$= \lim_{\Delta x\to 0^-}\frac{-1}{x(x+\Delta x)}$

$= \frac{-1}{x(x+0)} = -\frac{1}{x^2}$

17. $\lim_{x\to 3^-}f(x) = \lim_{x\to 3^-}\frac{x+2}{2} = \frac{5}{2}$

18. $\lim_{x\to 2^+}f(x) = \lim_{x\to 2^+}(-x^2+4x-2) = 2$

$\lim_{x\to 2^-}f(x) = \lim_{x\to 2^-}(x^2-4x+6) = 2$

$\lim_{x\to 2}f(x) = 2$

19. $\lim_{x\to 1^+}f(x) = \lim_{x\to 1^+}(x+1) = 2$

$\lim_{x\to 1^-}f(x) = \lim_{x\to 1^-}(x^3+1) = 2$

$\lim_{x\to 1}f(x) = 2$

20. $\lim_{x\to 1^+}f(x) = \lim_{x\to 1^+}(1-x) = 0$

21. $\lim_{x\to\pi/2}\sec x$ does not exist because

$\lim_{x\to(\pi/2)^+}\sec x$ and $\lim_{x\to(\pi/2)^-}\sec x$ do not exist.

22. $\lim_{x\to\pi}\cot x$ does not exist because

$\lim_{x\to\pi^+}\cot x$ and $\lim_{x\to\pi^-}\cot x$ do not exist.

23. $\lim_{x\to 2^+}(2x-[\![x]\!]) = 2(2)-2 = 2$

24. $\lim_{x\to 4^-}(5[\![x]\!] - 7) = 5(3) - 7 = 8$

$([\![x]\!] = 3 \text{ for } 3 \le x < 4)$

25. $\lim_{x\to 1}\left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right]\right) = 1 - (-1) = 2$

26. $\lim_{x\to 3}(2 - [\![-x]\!])$ does not exist because

$\lim_{x\to 3^-}(2 - [\![-x]\!]) = 2 - (-3) = 5$

and

$\lim_{x\to 3^+}(2 - [\![-x]\!]) = 2 - (-4) = 6.$

27. $f(x) = \dfrac{1}{x^2 - 4}$

has discontinuities at $x = -2$ and $x = 2$ because $f(-2)$ and $f(2)$ are not defined.

28. $f(x) = \dfrac{x^2 - 1}{x + 1}$

has a discontinuity at $x = -1$ because $f(-1)$ is not defined.

29. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$ has a discontinuity at

$x = 1$ because $f(1) = 2 \ne \lim_{x\to 1} f(x) = 1.$

30. $f(x) = \dfrac{[\![x]\!]}{2} + x$

has discontinuities at each integer k because $\lim_{x\to k^-} f(x) \ne \lim_{x\to k^+} f(x).$

31. $g(x) = \sqrt{49 - x^2}$ is continuous on $[-7, 7]$.

32. $f(t) = 3 - \sqrt{9 - t^2}$ is continuous on $[-3, 3]$.

33. $\lim_{x\to 0^-} f(x) = 3 = \lim_{x\to 0^+} f(x)$. f is continuous on $[-1, 4]$.

34. $g(2)$ is not defined. g is continuous on $[-1, 2)$.

35. $f(x) = \dfrac{6}{x}$ has a nonremovable discontinuity at $x = 0$.

36. $f(x) = \dfrac{3}{x - 2}$ has a nonremovable discontinuity at $x = 2$.

37. $f(x) = x^2 - 2x + 1$ is continuous for all real x.

38. $f(x) = x^2 - 9$ is continuous for all real x.

39. $f(x) = \dfrac{1}{x^2 + 1}$ is continuous for all real x.

40. $f(x) = \dfrac{1}{4 - x^2} = \dfrac{1}{(2 - x)(2 + x)}$ has nonremovable discontinuities at $x = \pm 2$ because $\lim_{x\to 2} f(x)$ and $\lim_{x\to -2} f(x)$ do not exist.

41. $f(x) = \cos\dfrac{\pi x}{2}$ is continuous for all real x.

42. $f(x) = 3x - \cos x$ is continuous for all real x.

43. $f(x) = \dfrac{x}{x^2 - 1}$ has nonremovable discontinuities at $x = 1$ and $x = -1$ because $\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$ do not exist.

44. $f(x) = \dfrac{x}{x^2 - x}$ is not continuous at $x = 0, 1$. Because $\dfrac{x}{x^2 - x} = \dfrac{1}{x - 1}$ for $x \neq 0$, $x = 0$ is a removable discontinuity, whereas $x = 1$ is a nonremovable discontinuity.

45. $f(x) = \dfrac{x-6}{x^2-36}$

has a nonremovable discontinuity at $x = -6$ because $\lim_{x\to -6} f(x)$ does not exist, and has a removable discontinuity at $x = 6$ because

$$\lim_{x\to 6} f(x) = \lim_{x\to 6}\frac{1}{x+6} = \frac{1}{12}.$$

46. $f(x) = \dfrac{x}{x^2+1}$ is continuous for all real x.

47. $f(x) = \dfrac{x-1}{(x+2)(x-1)}$

has a nonremovable discontinuity at $x = -2$ because $\lim_{x\to -2} f(x)$ does not exist, and has a removable discontinuity at $x = 1$ because

$$\lim_{x\to 1} f(x) = \lim_{x\to 1}\frac{1}{x+2} = \frac{1}{3}.$$

48. $f(x) = \dfrac{x+2}{(x+2)(x-5)}$

has a nonremovable discontinuity at $x = 5$ because $\lim_{x\to 5} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$$\lim_{x\to -2} f(x) = \lim_{x\to -2}\frac{1}{x-5} = -\frac{1}{7}.$$

49. $f(x) = \dfrac{|x-8|}{x-8}$ has a nonremovable discontinuity at $x = 8$ because $\lim_{x\to 8} f(x)$ does not exist.

50. $f(x) = \dfrac{|x+7|}{x+7}$

has a nonremovable discontinuity at $x = -7$ because $\lim_{x\to -7} f(x)$ does not exist.