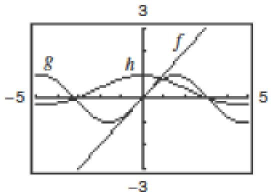
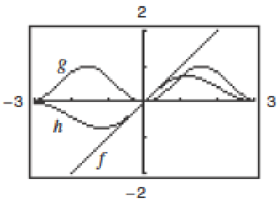
101. $f(x) = x,\ g(x) = \sin x,\ h(x) = \dfrac{\sin x}{x}$



When the x-values are "close to" 0 the magnitude of f is approximately equal to the magnitude of g. So, $|g|/|f| \approx 1$ when x is "close to" 0.

102. $f(x) = x,\ g(x) = \sin^2 x,\ h(x) = \dfrac{\sin^2 x}{x}$



When the x-values are "close to" 0 the magnitude of g is "smaller" than the magnitude of f and the magnitude of g is approaching zero "faster" than the magnitude of f. So, $|g|/|f| \approx 0$ when x is "close to" 0.

103. $s(t) = -16t^2 + 500$

$$\begin{aligned}\lim_{t\to 2}\frac{s(2) - s(t)}{2 - t} &= \lim_{t\to 2}\frac{-16(2)^2 + 500 - \left(-16t^2 + 500\right)}{2 - t}\\ &= \lim_{t\to 2}\frac{436 + 16t^2 - 500}{2 - t}\\ &= \lim_{t\to 2}\frac{16\left(t^2 - 4\right)}{2 - t}\\ &= \lim_{t\to 2}\frac{16(t - 2)(t + 2)}{2 - t}\\ &= \lim_{t\to 2} -16(t + 2) = -64 \text{ ft/sec}\end{aligned}$$

The wrench is falling at about 64 feet/second.

104. $s(t) = -16t^2 + 500 = 0$ when $t = \sqrt{\frac{500}{16}} = \frac{5\sqrt{5}}{2}$ sec. The velocity at time $a = \frac{5\sqrt{5}}{2}$ is

$$\lim_{t \to \left(\frac{5\sqrt{5}}{2}\right)} \frac{s\left(\frac{5\sqrt{5}}{2}\right) - s(t)}{\frac{5\sqrt{5}}{2} - t} = \lim_{t \to \left(\frac{5\sqrt{5}}{2}\right)} \frac{0 - \left(-16t^2 + 500\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t \to \left(\frac{5\sqrt{5}}{2}\right)} \frac{16\left(t^2 - \frac{125}{4}\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t \to \left(\frac{5\sqrt{5}}{2}\right)} \frac{16\left(t + \frac{5\sqrt{5}}{2}\right)\left(t - \frac{5\sqrt{5}}{2}\right)}{\frac{5\sqrt{5}}{2} - t}$$

$$= \lim_{t \to \frac{5\sqrt{5}}{2}} \left[-16\left(t + \frac{5\sqrt{5}}{2}\right)\right] = -80\sqrt{5} \text{ ft/sec}$$

$$\approx -178.9 \text{ ft/sec.}$$

The velocity of the wrench when it hits the ground is about 178.9 ft/sec.

105. $s(t) = -4.9t^2 + 200$

$$\lim_{t \to 3} \frac{s(3) - s(t)}{3 - t} = \lim_{t \to 3} \frac{-4.9(3)^2 + 200 - \left(-4.9t^2 + 200\right)}{3 - t}$$

$$= \lim_{t \to 3} \frac{4.9\left(t^2 - 9\right)}{3 - t}$$

$$= \lim_{t \to 3} \frac{4.9(t - 3)(t + 3)}{3 - t}$$

$$= \lim_{t \to 3} \left[-4.9(t + 3)\right]$$

$$= -29.4 \text{ m/sec}$$

The object is falling about 29.4 m/sec.

106. $-4.9t^2 + 200 = 0$ when $t = \sqrt{\frac{200}{4.9}} = \frac{20\sqrt{5}}{7}$ sec. The velocity at time $a = \frac{20\sqrt{5}}{7}$ is

$$\begin{aligned}
\lim_{t\to a}\frac{s(a)-s(t)}{a-t} &= \lim_{t\to a}\frac{0-\left[-4.9t^2+200\right]}{a-t} \\
&= \lim_{t\to a}\frac{4.9(t+a)(t-a)}{a-t} \\
&= \lim_{t\to \frac{20\sqrt{5}}{7}}\left[-4.9\left(t+\frac{20\sqrt{5}}{7}\right)\right] = -28\sqrt{5}\text{ m/sec} \\
&\approx -62.6\text{ m/sec.}
\end{aligned}$$

The velocity of the object when it hits the ground is about 62.6 m/sec.

107. Let $f(x) = 1/x$ and $g(x) = -1/x$. $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$$\lim_{x\to 0}\left[f(x)+g(x)\right] = \lim_{x\to 0}\left[\frac{1}{x}+\left(-\frac{1}{x}\right)\right] = \lim_{x\to 0}[0] = 0$$

and therefore does not exist.

108. Suppose, on the contrary, that $\lim_{x\to c} g(x)$ exists. Then, because $\lim_{x\to c} f(x)$ exists, so would $\lim_{x\to c}\left[f(x)+g(x)\right]$, which is a contradiction. So, $\lim_{x\to c} g(x)$ does not exist.

109. Given $f(x) = b$, show that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $\left|f(x) - b\right| < \varepsilon$ whenever $\left|x - c\right| < \delta$. Because $\left|f(x) - b\right| = \left|b - b\right| = 0 < \varepsilon$ for every $\varepsilon > 0$, any value of $\delta > 0$ will work.

110. Given $f(x) = x^n$, n is a positive integer, then

$$\begin{aligned}
\lim_{x\to c} x^n &= \lim_{x\to c}\left(xx^{n-1}\right) \\
&= \left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-1}\right] = c\left[\lim_{x\to c}\left(xx^{n-2}\right)\right] \\
&= c\left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-2}\right] = c(c)\lim_{x\to c}\left(xx^{n-3}\right) \\
&= \cdots = c^n.
\end{aligned}$$

111. If $b = 0$, the property is true because both sides are equal to 0. If $b \neq 0$, let $\varepsilon > 0$ be given. Because $\lim_{x \to c} f(x) = L$, there exists $\delta > 0$ such that

$\left|f(x) - L\right| < \varepsilon/|b|$ whenever $0 < |x - c| < \delta$. So, whenever $0 < |x - c| < \delta$, we have

$$|b|\left|f(x) - L\right| < \varepsilon \quad \text{or} \quad \left|bf(x) - bL\right| < \varepsilon$$

which implies that $\lim_{x \to c}\left[bf(x)\right] = bL$.

112. Given $\lim_{x \to c} f(x) = 0$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\left|f(x) - 0\right| < \varepsilon$ whenever $0 < |x - c| < \delta$.

Now $\left|f(x) - 0\right| = \left|f(x)\right| = \left|\left|f(x)\right| - 0\right| < \varepsilon$ for $|x - c| < \delta$. Therefore, $\lim_{x \to c}\left|f(x)\right| = 0$.

113.

$$-M\left|f(x)\right| \leq f(x)g(x) \leq M\left|f(x)\right|$$

$$\lim_{x \to c}\left(-M\left|f(x)\right|\right) \leq \lim_{x \to c} f(x)g(x) \leq \lim_{x \to c}\left(M\left|f(x)\right|\right)$$

$$-M(0) \leq \lim_{x \to c} f(x)g(x) \leq M(0)$$

$$0 \leq \lim_{x \to c} f(x)g(x) \leq 0$$

Therefore, $\lim_{x \to c} f(x)g(x) = 0$.

114. (a) If $\lim_{x \to c}\left|f(x)\right| = 0$, then $\lim_{x \to c}\left[-\left|f(x)\right|\right] = 0$.

$$-\left|f(x)\right| \leq f(x) \leq \left|f(x)\right|$$

$$\lim_{x \to c}\left[-\left|f(x)\right|\right] \leq \lim_{x \to c} f(x) \leq \lim_{x \to c}\left|f(x)\right|$$

$$0 \leq \lim_{x \to c} f(x) \leq 0$$

Therefore, $\lim_{x \to c} f(x) = 0$.

(b) Given $\lim_{x \to c} f(x) = L$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\left|f(x) - L\right| < \varepsilon$ whenever $0 < |x - c| < \delta$. Since $\left|\left|f(x)\right| - |L|\right| \leq \left|f(x) - L\right| < \varepsilon$ for $|x - c| < \delta$, then $\lim_{x \to c}\left|f(x)\right| = |L|$.

115. Let

$$f(x) = \begin{cases} 4, & \text{if } x \geq 0 \\ -4, & \text{if } x < 0 \end{cases}$$

$\lim_{x\to 0}|f(x)| = \lim_{x\to 0} 4 = 4.$

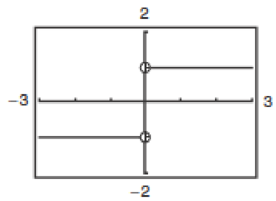
$\lim_{x\to 0} f(x)$ does not exist because for

$x < 0,\ f(x) = -4$ and for $x \geq 0,\ f(x) = 4.$

116. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = \lim_{x\to 2} f(x) = 3$

The value of f at $x = 2$ is irrelevant.

117. The limit does not exist because the function approaches 1 from the right side of 0 and approaches −1 from the left side of 0.



118. False. $\lim_{x\to\pi} \frac{\sin x}{x} = \frac{0}{\pi} = 0$

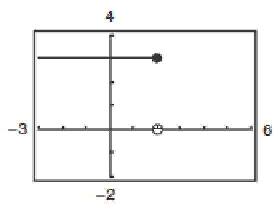
119. True.

120. False. Let

$$f(x) = \begin{cases} x & x \neq 1 \\ 3 & x = 1 \end{cases}, \quad c = 1.$$

Then $\lim_{x\to 1} f(x) = 1$ but $f(1) \neq 1.$

121. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.



122. False. Let

$f(x) = \frac{1}{2}x^2$ and $g(x) = x^2.$

Then $f(x) < g(x)$ for all $x \neq 0.$ But

$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0.$

123. $\lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x}$$

$$= \left[\lim_{x\to 0}\frac{\sin x}{x}\right]\left[\lim_{x\to 0}\frac{\sin x}{1+\cos x}\right]$$

$$= (1)(0) = 0$$

124. $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$

$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$

$\lim_{x\to 0} f(x)$ does not exist.

No matter how "close to" 0 x is, there are still an infinite number of rational and irrational numbers so that $\lim_{x\to 0} f(x)$ does not exist.

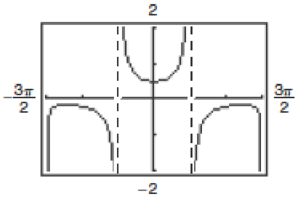
$\lim_{x\to 0} g(x) = 0$

when x is "close to" 0, both parts of the function are "close to" 0.

125. $f(x) = \dfrac{\sec x - 1}{x^2}$

(a) The domain of f is all $x \neq 0, \pi/2 + n\pi$.

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent.

(c) $\lim_{x\to 0} f(x) = \dfrac{1}{2}$

(d) $$\frac{\sec x - 1}{x^2} = \frac{\sec x - 1}{x^2}\cdot\frac{\sec x + 1}{\sec x + 1} = \frac{\sec^2 x - 1}{x^2(\sec x + 1)}$$

$$= \frac{\tan^2 x}{x^2(\sec x + 1)} = \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1}$$

So, $$\lim_{x\to 0}\frac{\sec x - 1}{x^2} = \lim_{x\to 0}\frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1}$$

$$= 1(1)\left(\frac{1}{2}\right) = \frac{1}{2}.$$

126. (a) $$\lim_{x\to 0}\frac{1 - \cos x}{x^2} = \lim_{x\to 0}\frac{1 - \cos x}{x^2}\cdot\frac{1 + \cos x}{1 + \cos x}$$

$$= \lim_{x\to 0}\frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\frac{1}{1 + \cos x}$$

$$= (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

(b) From part (a),

$$\frac{1 - \cos x}{x^2} \approx \frac{1}{2} \Rightarrow 1 - \cos x \approx \frac{1}{2}x^2 \Rightarrow \cos x \approx 1 - \frac{1}{2}x^2 \text{ for } x \approx 0.$$

(c) $\cos(0.1) \approx 1 - \dfrac{1}{2}(0.1)^2 = 0.995$

(d) $\cos(0.1) \approx 0.9950$, which agrees with part (c).

127. The graphing utility was set in degree mode, instead of *radian* mode.