1.

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.2041	0.2004	0.2000	0.2000	0.1996	0.1961

$\lim_{x \to 4} \frac{x - 4}{x^2 - 3x - 4} \approx 0.2000$ $\left(\text{Actual limit is } \frac{1}{5}.\right)$

2.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

$\lim_{x \to 2} \frac{x - 2}{x^2 - 4} \approx 0.25$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$

3.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.2050	0.2042	0.2041	0.2041	0.2040	0.2033

$\lim_{x \to 0} \frac{\sqrt{x + 6} - \sqrt{6}}{x} \approx 0.2041$ $\left(\text{Actual limit is } \frac{1}{2\sqrt{6}}.\right)$

4.

x	–5.1	–5.01	–5.001	–4.999	–4.99	–4.9
$f(x)$	–0.1662	–0.1666	–0.1667	–0.1667	–0.1667	–0.1671

$\lim_{x \to -5} \frac{\sqrt{4 - x} - 3}{x + 5} \approx -0.1667$ $\left(\text{Actual limit is } -\frac{1}{6}.\right)$

5.

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	–0.0641	–0.0627	–0.0625	–0.0625	–0.0623	–0.0610

$\lim_{x \to 3} \frac{[1/(x + 1)] - (1/4)}{x - 3} \approx -0.0625$ $\left(\text{Actual limit is } -\frac{1}{16}.\right)$

6.

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.0408	0.0401	0.0400	0.0400	0.0399	0.0392

$\lim_{x \to 4} \frac{[x/(x + 1)] - (4/5)}{x - 4} \approx 0.04$ $\left(\text{Actual limit is } \frac{1}{25}.\right)$

7.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.9983	0.99998	1.0000	1.0000	0.99998	0.9983

$\lim_{x \to 0} \frac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.) (Make sure you use radian mode.)

8.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.0500	0.0050	0.0005	−0.0005	−0.0050	−0.0500

$\lim_{x \to 0} \frac{\cos x - 1}{x} \approx 0.0000$ (Actual limit is 0.) (Make sure you use radian mode.)

9.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

$\lim_{x \to 1} \frac{x - 2}{x^2 + x - 6} \approx 0.2500$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$

10.

x	−3.1	−3.01	−3.001	−2.999	−2.99	−2.9
$f(x)$	1.1111	1.0101	1.0010	0.9990	0.9901	0.9091

$\lim_{x \to -3} \frac{x + 3}{x^2 + 7x + 12} \approx 1.0000$ (Actual limit is 1.)

11.

x	−2.1	−2.01	−2.001	−1.999	−1.99	−1.9
$f(x)$	12.6100	12.0601	12.0060	11.9940	11.9401	11.4100

$\lim_{x \to -2} \frac{x^3 + 8}{x + 2} \approx 12.0000$ (Actual limit is 12.)

12.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.7340	0.6733	0.6673	0.6660	0.6600	0.6015

$\lim_{x \to 1} \frac{x^4 - 1}{x^6 - 1} \approx 0.6666$ $\left(\text{Actual limit is } \frac{2}{3}.\right)$

13.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	0.4950	0.5000	0.5000	0.5000	0.5000	0.4950

$\lim_{x \to 0} \frac{\tan x}{\tan 2x} \approx 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

14.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	1.9867	1.9999	2.0000	2.0000	1.9999	1.9867

$\lim_{x \to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.) (Make sure you use radian mode.)

15. $\lim_{x \to 3} (4 - x) = 1$

16. $\lim_{x \to 1} (x^2 + 3) = 4$

17. $\lim_{x \to 1} f(x) = \lim_{x \to 1} (x^2 + 3) = 4$

18. $\lim_{x \to 2} f(x) = \lim_{x \to 2} (4 - x) = 2$

19. $\lim_{x \to 2} \frac{|x - 2|}{x - 2}$ does not exist.

For values of x to the left of 2, $\frac{|x - 2|}{(x - 2)} = -1$, whereas

for values of x to the right of 2, $\frac{|x - 2|}{(x - 2)} = 1$.

20. $\lim_{x \to 5} \frac{2}{x - 5}$ does not exist because the function increases and decreases without bound as x approaches 5.

21. $\lim_{x \to 1} \sin \pi x = 0$

22. $\lim_{x \to 0} \sec x = 1$

23. $\lim_{x \to 0} \cos(1/x)$ does not exist because the function oscillates between -1 and 1 as x approaches 0.

24. $\lim_{x \to \pi/2} \tan x$ does not exist because the function increases without bound as x approaches $\frac{\pi}{2}$ from the left and decreases without bound as x approaches $\frac{\pi}{2}$ from the right.

25. (a) $f(1)$ exists. The black dot at (1, 2) indicates that $f(1) = 2$.

(b) $\lim_{x \to 1} f(x)$ does not exist. As x approaches 1 from the left, $f(x)$ approaches 3.5, whereas as x approaches 1 from the right, $f(x)$ approaches 1.

(c) $f(4)$ does not exist. The hollow circle at $(4, 2)$ indicates that f is not defined at 4.

(d) $\lim_{x \to 4} f(x)$ exists. As x approaches 4, $f(x)$ approaches 2: $\lim_{x \to 4} f(x) = 2$.

26. (a) $f(-2)$ does not exist. The vertical dotted line indicates that f is not defined at –2.

(b) $\lim_{x \to -2} f(x)$ does not exist. As x approaches –2, the values of $f(x)$ do not approach a specific number.

(c) $f(0)$ exists. The black dot at $(0, 4)$ indicates that $f(0) = 4$.

(d) $\lim_{x \to 0} f(x)$ does not exist. As x approaches 0 from the left, $f(x)$ approaches $\frac{1}{2}$, whereas as x approaches 0 from the right, $f(x)$ approaches 4.

(e) $f(2)$ does not exist. The hollow circle at $\left(2, \frac{1}{2}\right)$ indicates that $f(2)$ is not defined.

(f) $\lim_{x \to 2} f(x)$ exists. As x approaches 2, $f(x)$ approaches $\frac{1}{2}$: $\lim_{x \to 2} f(x) = \frac{1}{2}$.

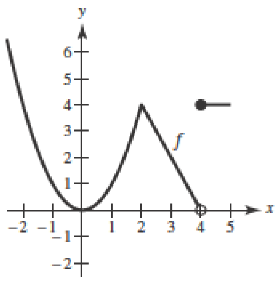
(g) $f(4)$ exists. The black dot at $(4, 2)$ indicates that $f(4) = 2$.

(h) $\lim_{x \to 4} f(x)$ does not exist. As x approaches 4, the values of $f(x)$ do not approach a specific number.

27. $\lim_{x \to c} f(x)$ exists for all $c \neq -3$.

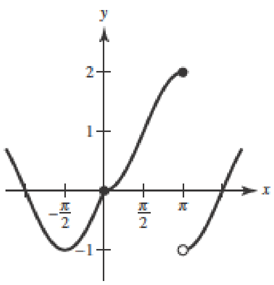
28. $\lim_{x \to c} f(x)$ exists for all $c \neq -2, 0$.

29.



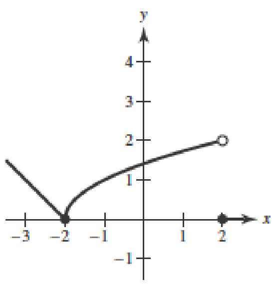
$\lim_{x \to c} f(x)$ exists for all values of $c \neq 4$.

30.

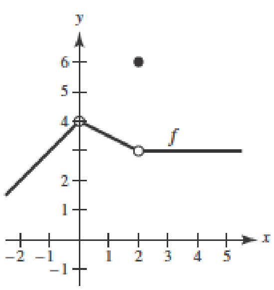


$\lim_{x \to c} f(x)$ exists for all values of $c \neq \pi$.

31. One possible answer is

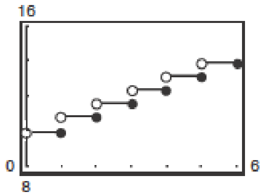


32. One possible answer is



33. $C(t) = 9.99 - 0.79[\![-(t - 1)]\!]$

(a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	11.57	12.36	12.36	12.36	12.36	12.36	12.36

$\lim_{t \to 3.5} C(t) = 12.36$

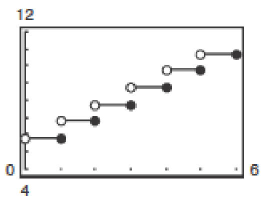
(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	10.78	11.57	11.57	11.57	12.36	12.36	12.36

The $\lim_{t \to 3} C(t)$ does not exist because the values of C approach different values as t approaches 3 from both sides.

34. $C(t) = 5.79 - 0.99[\![-(t - 1)]\!]$

(a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	7.77	8.76	8.76	8.76	8.76	8.76	8.76

$\lim_{t \to 3.5} C(t) = 8.76$

(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	6.78	7.77	7.77	7.77	8.76	8.76	8.76

The limit $\lim_{t \to 3} C(t)$ does not exist because the values of C approach different values as t approaches 3 from both sides.

35. You need $|f(x) - 3| = |(x + 1) - 3| = |x - 2| < 0.4$. So, take $\delta = 0.4$. If $0 < |x - 2| < 0.4$, then $|x - 2| = |(x + 1) - 3| = |f(x) - 3| < 0.4$, as desired.

36. You need $|f(x) - 1| = \left|\frac{1}{x-1} - 1\right| = \left|\frac{2-x}{x-1}\right| < 0.01$. Let $\delta = \frac{1}{101}$. If $0 < |x - 2| < \frac{1}{101}$, then

$$-\frac{1}{101} < x - 2 < \frac{1}{101} \Rightarrow 1 - \frac{1}{101} < x - 1 < 1 + \frac{1}{101}$$
$$\Rightarrow \frac{100}{101} < x - 1 < \frac{102}{101}$$
$$\Rightarrow |x - 1| > \frac{100}{101}$$

and you have

$$|f(x) - 1| = \left|\frac{1}{x-1} - 1\right| = \left|\frac{2-x}{x-1}\right| < \frac{1/101}{100/101} = \frac{1}{100} = 0.01.$$

37. You need to find δ such that $0 < |x - 1| < \delta$ implies

$|f(x) - 1| = \left|\frac{1}{x} - 1\right| < 0.1$. That is,

$$-0.1 < \frac{1}{x} - 1 < 0.1$$
$$1 - 0.1 < \frac{1}{x} < 1 + 0.1$$
$$\frac{9}{10} < \frac{1}{x} < \frac{11}{10}$$
$$\frac{10}{9} > x > \frac{10}{11}$$
$$\frac{10}{9} - 1 > x - 1 > \frac{10}{11} - 1$$
$$\frac{1}{9} > x - 1 > -\frac{1}{11}.$$

So take $\delta = \frac{1}{11}$. Then $0 < |x - 1| < \delta$ implies

$$-\frac{1}{11} < x - 1 < \frac{1}{11}$$
$$-\frac{1}{11} < x - 1 < \frac{1}{9}.$$

Using the first series of equivalent inequalities, you obtain

$$|f(x) - 1| = \left|\frac{1}{x} - 1\right| < 0.1.$$

38. You need to find δ such that $0 < |x - 2| < \delta$ implies

$|f(x) - 3| = |x^2 - 1 - 3| = |x^2 - 4| < 0.2$. That is,

$$-0.2 < x^2 - 4 < 0.2$$
$$4 - 0.2 < x^2 < 4 + 0.2$$
$$3.8 < x^2 < 4.2$$
$$\sqrt{3.8} < x < \sqrt{4.2}$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2$$

So take $\delta = \sqrt{4.2} - 2 \approx 0.0494$.

Then $0 < |x - 2| < \delta$ implies

$$-\left(\sqrt{4.2} - 2\right) < x - 2 < \sqrt{4.2} - 2$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2.$$

Using the first series of equivalent inequalities, you obtain

$$|f(x) - 3| = |x^2 - 4| < 0.2.$$

39. $\lim\limits_{x \to 2}(3x + 2) = 8 = L$

$$|(3x + 2) - 8| < 0.01$$
$$|3x - 6| < 0.01$$
$$3|x - 2| < 0.01$$
$$0 < |x - 2| < \frac{0.01}{3} \approx 0.0033 = \delta$$

So, if $0 < |x - 2| < \delta = \frac{0.01}{3}$, you have

$$3|x - 2| < 0.01$$
$$|3x - 6| < 0.01$$
$$|(3x + 2) - 8| < 0.01$$
$$|f(x) - L| < 0.01.$$

40. $\lim_{x\to 4}\left(4 - \frac{x}{2}\right) = 2 = L$

$$\left|\left(4 - \frac{x}{2}\right) - 2\right| < 0.01$$

$$\left|2 - \frac{x}{2}\right| < 0.01$$

$$\left|-\frac{1}{2}(x - 4)\right| < 0.01$$

$$0 < \left|x - 4\right| < 0.02 = \delta$$

Hence, if $0 < \left|x - 4\right| < \delta = 0.02$, you have

$$\left|-\frac{1}{2}(x - 4)\right| < 0.01$$

$$\left|2 - \frac{x}{2}\right| < 0.01$$

$$\left|\left(4 - \frac{x}{2}\right) - 2\right| < 0.01$$

$$\left|f(x) - L\right| < 0.01.$$

41. $\lim_{x\to 5}\left(x^2 + 4\right) = 29 = L$

$$\left|\left(x^2 + 4\right) - 29\right| < 0.01$$

$$\left|x^2 - 25\right| < 0.01$$

$$\left|(x + 5)(x - 5)\right| < 0.01$$

$$\left|x - 5\right| < \frac{0.01}{\left|x + 5\right|}$$

If you assume $4 < x < 6$, then $\delta = 0.01/11 \approx 0.0009$.

So, if $0 < \left|x - 5\right| < \delta = \frac{0.01}{11}$, you have

$$\left|x - 5\right| < \frac{0.01}{11} < \frac{1}{\left|x + 5\right|}(0.01)$$

$$\left|x - 5\right|\left|x + 5\right| < 0.01$$

$$\left|x^2 - 25\right| < 0.01$$

$$\left|\left(x^2 + 4\right) - 29\right| < 0.01$$

$$\left|f(x) - L\right| < 0.01.$$

42. $\lim_{x\to 2}(x^2 - 3) = 1 = L$

$$\left|(x^2 - 3) - 1\right| < 0.01$$

$$\left|x^2 - 4\right| < 0.01$$

$$\left|(x + 2)(x - 2)\right| < 0.01$$

$$\left|x + 2\right|\left|x - 2\right| < 0.01$$

$$\left|x - 2\right| < \frac{0.01}{\left|x + 2\right|}$$

If you assume $1 < x < 3$, then $\delta = 0.01/5 = 0.002$.

So, if $0 < \left|x - 2\right| < \delta = 0.002$, you have

$$\left|x - 2\right| < 0.002 = \frac{1}{5}(0.01) < \frac{1}{\left|x + 2\right|}(0.01)$$

$$\left|x + 2\right|\left|x - 2\right| < 0.01$$

$$\left|x^2 - 4\right| < 0.01$$

$$\left|(x^2 - 3) - 1\right| < 0.01$$

$$\left|f(x) - L\right| < 0.01.$$

43. $\lim_{x\to 4}(x + 2) = 6$

Given $\varepsilon > 0$:

$$\left|(x + 2) - 6\right| < \varepsilon$$

$$\left|x - 4\right| < \varepsilon = \delta$$

So, let $\delta = \varepsilon$. So, if $0 < \left|x - 4\right| < \delta = \varepsilon$, you have

$$\left|x - 4\right| < \varepsilon$$

$$\left|(x + 2) - 6\right| < \varepsilon$$

$$\left|f(x) - L\right| < \varepsilon.$$

44. $\lim_{x \to -3} (2x + 5) = -1$

Given $\varepsilon > 0$:

$$\left|(2x + 5) - (-1)\right| < \varepsilon$$
$$\left|2x + 6\right| < \varepsilon$$
$$2\left|x + 3\right| < \varepsilon$$
$$\left|x + 3\right| < \frac{\varepsilon}{2} = \delta$$

So, let $\delta = \varepsilon/2$.

So, if $0 < \left|x + 3\right| < \delta = \frac{\varepsilon}{2}$, you have

$$\left|x + 3\right| < \frac{\varepsilon}{2}$$
$$\left|2x + 6\right| < \varepsilon$$
$$\left|(2x + 5) - (-1)\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

45. $\lim_{x \to 1} \left(\frac{2}{5}x + 7\right) = \frac{2}{5}(1) + 7 = \frac{37}{5}$

Given $\varepsilon > 0$:

$$\left|\left(\tfrac{2}{5}x + 7\right) - \tfrac{37}{5}\right| = \left|\tfrac{2}{5}x - \tfrac{2}{5}\right| < \varepsilon$$
$$\tfrac{2}{5}\left|x - 1\right| < \varepsilon$$
$$\left|x - 1\right| < \tfrac{5}{2}\varepsilon$$

So, let $\delta = \frac{5}{2}\varepsilon$.

So, if $0 < \left|x - 1\right| < \delta = \frac{5}{2}\varepsilon$, you have

$$\left|x - 1\right| < \tfrac{5}{2}\varepsilon$$
$$\left|\tfrac{2}{5}x - \tfrac{2}{5}\right| < \varepsilon$$
$$\left|\left(\tfrac{2}{5}x + 7\right) - \tfrac{37}{5}\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

46. $\lim_{x\to -4}\left(\frac{1}{2}x - 1\right) = \frac{1}{2}(-4) - 1 = -3$

Given $\varepsilon > 0$:

$$\left|\left(\tfrac{1}{2}x - 1\right) - (-3)\right| < \varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\tfrac{1}{2}\left|x - (-4)\right| < \varepsilon$$
$$\left|x - (-4)\right| < 2\varepsilon$$

So, let $\delta = 2\varepsilon$.

So, if $0 < \left|x - (-4)\right| < \delta = 2\varepsilon$, you have

$$\left|x - (-4)\right| < 2\varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\left|\left(\tfrac{1}{2}x - 1\right) + 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

47. $\lim_{x\to 2}(-1) = -1$

Given $\varepsilon > 0$: $\left|-1 - (-1)\right| < \varepsilon$

$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|(-1) - (-1)\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

48. $\lim_{x\to 6} 3 = 3$

Given $\varepsilon > 0$:

$$\left|3 - 3\right| < \varepsilon$$
$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|3 - 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

49. $\lim_{x \to 4} \sqrt{x} = \sqrt{4} = 2$

Given $\varepsilon > 0$: $\left|\sqrt{x} - 2\right| < \varepsilon$

$$\left|\sqrt{x} - 2\right|\left|\sqrt{x} + 2\right| < \varepsilon\left|\sqrt{x} + 2\right|$$

$$\left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$

Assuming $1 < x < 9$, you can choose $\delta = 3\varepsilon$. Then,

$$0 < \left|x - 4\right| < \delta = 3\varepsilon \Rightarrow \left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$

$$\Rightarrow \left|\sqrt{x} - 2\right| < \varepsilon.$$

50. $\lim_{x \to 0} \sqrt[3]{x} = 0$

Given $\varepsilon > 0$: $\left|\sqrt[3]{x} - 0\right| < \varepsilon$

$$\left|\sqrt[3]{x}\right| < \varepsilon$$

$$\left|x\right| < \varepsilon^3 = \delta$$

So, let $\delta = \varepsilon^3$.

So, for $0\left|x - 0\right|\delta = \varepsilon^3$, you have

$$\left|x\right| < \varepsilon^3$$

$$\left|\sqrt[3]{x}\right| < \varepsilon$$

$$\left|\sqrt[3]{x} - 0\right| < \varepsilon$$

$$\left|f(x) - L\right| < \varepsilon.$$